Particles Here and Beyond the Mirror

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This study looks at motion of particles using mathematical methods of chronometric invariants (physical observable values in General Relativity). It is shown that aside for mass-bearing particles and light-like particles "zero-particles" can exist in fully degenerated space-time (zero-space). For a regular observer zero-particles move instantly, thus transferring long-range action. Further we show existence of two separate areas in unhomogeneous space-time, where observable time flows into future and into past, while this duality is not found in homogeneous space-time. These areas are referred to as our world, where time flows into future and as the mirror Universe, where time flows in past. The areas are separated with a space-time membrane, referred to as zero-space, where observable time stops.

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Foreword

This book is the generalization of our manuscripts of 1991–1996. These papers were used as the basis for further studies in 1997–99, which were published in our *Fields, Vacuum, and the Mirror Universe*[1] in March, 2001 as a contemporary supplement to the famous *The Classical Theory of Fields* by L. D. Landau and E. M. Lifshitz. After this book reached the bookshelves some of our readers began asking for a detailed account on the previous studies. To meet their request we prepared our manuscripts of 1991–96 as a book with some recent amendments. The result is given here for your consideration. The book is advanced by the transcript of a public lecture that one of the authors gave in St. Petersburg in May, 2001. The lecture contains an account of the history and main results of our studies in a broad accessible form.

In conclusion we would like to express hearty thanks to our parents and whole family for permanent support. Belated thanks go to Dr. Abram Zelmanov (1913–1987) and Prof. Kyril Stanyukovich (1916–1989), our teachers, for years of friendly discussions. Also we highly appreciate contribution from our colleague Dr. Boris Levin, who supported our ideas over the recent years. We are grateful to Dr. B. A. Kotov, Prof. V. I. Sokolov, Prof. E. K. Iordanishvili, and Dr. I. A. Kotova, for friendly discussion and for good questions that improved the manuscript greatly. Many thanks go to Grigory Semyonov, a friend of ours, for preparing the manuscript in English. Also we are grateful to our publisher Domingo Marín Ricoy for his interest to our works. Specially we are thankful to Dr. Basil K. Malyshev who provided his $\mathcal{B}_\mathcal{A}\mathcal{K}_\mathcal{O}\mathcal{M}_\mathcal{A}$-TEX system[2] for us.

D. D. Rabounski and L. B. Borissova

Dear Ladies and Gentlemen,

In this lecture I would like to present for your kind attention the history of our research and some of the results we have obtained over the recent decade. I am going to focus largely on two subjects. The first is zero-space, zero-particles and possible instant displacement in space. The second is explanation of anomalous annihilation of orthopositronium, which was first observed in 1987, by interaction of our world (world of positive energy of particles) with the mirror Universe (where particles bear negative energy).

The General Theory of Relativity is built upon view of the world as a four-dimensional space-time, where any and all objects possess not three-dimensional volume alone, but their "longitude" in time. That is, any physical body, including ours, is a really existing four-dimensional instance with the shape of a cylinder elongated in time (cylinder of events of this body), created by perplexion of other cylinders at the moment of its "birth" and split into many other ones at the moment of its "death". For example, for a human the "temporal length" is the duration of their life from conception till death.

The view of the world as a space-time continuum takes its origin from the historical report by Herman Minkowski, which he gave on 21 September 1908 in Cologne. There he presented a geometric interpretation of the principle of invariance of light speed and of Lorentz transformations. Later Albert Einstein came to the idea that became the cornerstone of his General Relativity (1908–1916). That was the "geometric concept of the world", according to which geometric structure of space-time defines all properties of the Universe (both in its observed areas and in areas inaccessible for observations).

What is space is intuitively clear. But what kind of space, out of those known in the contemporary mathematics, can comprise three-dimensional space and time into the single manifold?

Consideration of the problem led Einstein to the fact that the only way to represent space-time in the way that fits the existing experimental data is given by *four-dimensional pseudo-Riemannian space.* This is a partial case of the family of Riemannian spaces, i. e. spaces which geometry is Riemannian (the square of distance ds^2 between infinitely close points is defined by metric $ds^2{=}g_{\alpha\beta}dx^\alpha dx^\beta{=}inv$). In Riemannian space coordinate axes can be of any kind. Four-dimensional pseudo-Riemannian space is different on the account of the fact that here there is principal difference between the three-dimensional

[1]Borissova L. B. and Rabounski D. D. Fields, vacuum, and the mirror Universe. Editorial URSS, Moscow, 2001 (the book also is available: *Mathematical Physics Preprint Archive*, mp_arc: 03-206; *CERN Document Server*, EXT–2003–025).

[2]http://www.tex.ac.uk/tex-archive/systems/win32/bakoma/

space, perceived as space, from the fourth axis — time. From mathematical viewpoint this looks as follows: three spatial axes are real, while the time axis is imaginary (or vice versa), and choice of such conditions is arbitrary.

A partial case of flat, uniform, and isotropic pseudo-Riemannian space is referred to as *Minkowski space*. This is the space-time of Special Relativity, an abstracted case where gravitational field, space's rotation or deformation are absent. In the general case the real pseudo-Riemannian space is curved, non-uniform and anisotropic. This is General Relativity space-time where gravitational field, rotation and deformation are present.

Numerous experiments to verify General Relativity showed that four-dimensional pseudo-Riemannian space is our basic space-time, within the high precision of up-to-date measurements.

But very soon after A. Eddington in 1919 gave the first proof that Sun rays are curved by its gravitational field, many researchers faced strong obstacles in fitting together calculations made in the frame of General Relativity with existing results of observations and experiments.

Here is the problem in a nutshell. All equations in General Relativity are put down in so-called *general covariant form*, which does not depend upon choice of frame of reference in space-time. The equations, as well as the variables they contain, are four-dimensional. Which of those four-dimensional variables are observable in real physical experiments, i. e. which components are physical observable values?

Intuitively we may assume that three-dimensional components of four-dimensional tensor values are observable. But we can not be absolutely sure that what we observe are three-dimensional components *per se*, not more complicated variables which depend from other factors (e. g. from properties of physical standards of the space of reference).

Besides, what is the temporal component of a four-dimensional value? In certain cases this is evident. For instance, three-dimensional components of four-dimensional potential of electromagnetic field A^α (Greek indices are four-dimensional, Roman ones are three-dimensional) make up three-dimensional vector potential A^i, while the zero (temporal) component $A^0{=}\varphi$ is the scalar field potential. But this is not true for any given vector field.

Further, four-dimensional vector (1st rank tensor) has as few as 4 components (one temporal and three spatial). A 2nd rank tensor, e. g. rotation or deformation tensor, has 16 components: one temporal, nine spatial ones and six mixed (time-space) ones. Tensor of higher ranks have even more components. Are mixed components physical observable values? This is another question that had no definite answer.

We see that to distinguish physical observable values in General Relativity is not a trivial problem. Ideally we would like to have a mathematical technique to calculate physical observable values for tensors of any given ranks *unambiguously*.

Numerous attempts to develop such mathematical technique to define physical observable values were made in 1930s by outstanding researchers of that time. The goal was nearly attained by L. D. Landau and E. M. Lifshitz in their *The Classical Theory of Fields*, first published in Russian in 1939. Aside for discussing the problem of physical observable values itself, they introduced interval of physical observable time and three-dimensional observable interval, which depend upon the properties of the space of reference and observer's physical standards. But all attempts made in 1930s were limited to solving certain particular problems. None of them led to development of a versatile mathematical apparatus.

Such apparatus was developed by A. L. Zelmanov in 1941–44, who called it *theory of chronometric invariants*. Zelmanov first published it in his dissertation thesis[3].

The essence of Zelmanov's method is simple. As known, components of a tensor value are defined in so-called *ortho-frame*, a system of orthogonal ideal (straight and uniform) axes, which are tangential to real physical axes (curved and non-uniform) at the zero point of reference[4]. Hence in a real laboratory

[3]Zelmanov A. L. On deformation and curvature of accompanying space. Dissertation thesis, v. 1 and v. 2. Sternberg Astron. Institute, Moscow, 1944 (*in Russian*).

[4]From mathematical viewpoint in every point of real space-time we choose a tangential ideal pseudo-Euclidean space. In other words, components of the tensor are chosen in a tangential pseudo-Euclidean space, but not real curved and unhomogeneous pseudo-Riemannian space.

we do not measure components of a four-dimensional value, bit rather its *projections* on curved, non-uniform coordinate axes of a real space of reference.

Real space-time can be imagined as a set of curved and non-uniform spatial sections, "stacked" on the temporal axis (which is non-uniform, too). Hence a four-dimensional value is projected onto observer's time and their spatial section. From technical viewpoint projecting onto time and space is done by means of special tensors (*operators of projection*), which were introduced by Zelmanov and have all properties of operators of projections in a pseudo-Riemannian space. As a result we obtain physical observable projections of a four-dimensional value.

Because projection is done in curved and non-uniform structure of real space, the result depends upon properties of observer's space. Zelmanov proved that the features of space, which possess properties of physical observable values (observable projections), are gravitational inertial force, rotation of space and its deformation. In other words, physical observable values depend upon gravitational inertial force, rotation and deformation of observer's space.

Numerous experiments, which have been done since 1950s in various countries, showed significant impact of space's properties on the measured length and observed time. The most tremendous out of those experiments were no-landing flights around the globe in October 1971. During a flight along the Earth's rotation, observer's space on the plane had more rotation than the space of the observer who stayed fixed on the ground. During a flight against the Earth's rotation it was vice versa. Atomic clock on the plane showed significant variation of observed time depending upon the speed of space's rotation[5].

The only reason to take such an extended introduction was to emphasize that all our results were obtained using Zelmanov's method of physical observable values. As you will see later, regular general covariant methods could not lead to such results.

To summarize, we can say that observable values in real curved, non-uniform and anisotropic space differ dramatically from so-called "coordinate values", which are defined in an ideal space. This presented two cases, which could not be studied using general covariant methods: (a) "split" of the regular space-time into an area, where time flows from past into future (our world) and an area, where time flows into the opposite direction (the mirror Universe); (b) existence of an area of space-time, where four-dimensional interval, observable three-dimensional interval and interval of observable time are zeroes (zero-space).

Let us discuss the first case first. According to the theory of chronometric invariants, interval of physical observable time (projection of four-dimensional increment dx^α onto time axis) is the value $d\tau=\left(1-\frac{w}{c^2}\right)dt-\frac{1}{c^2}v_i dx^i$, i. e. it depends upon gravitational potential w, upon velocity of space rotation v_i and upon the point of observation (three-dimensional coordinates x^i). We will refer to the value $\frac{d\tau}{dt}$ as *function of observer's time*, i. e. it shows where the observer themselves moves in respect to the time axis $x^0=ct$. Hence, at least three different areas exist, where the observer's function of time bears different sign. Each of these areas is populated by particles of a specific type:

1. in the area where $\frac{d\tau}{dt}>0$ particles move into future in respect to the axis of time coordinate (direct flow of time). This is our world. It is inhabited by mass-bearing particles (both real sub-light-speed ones and imaginary tachyons) with positive relativistic masses, as well as massless (light-like) particles with positive own frequency. In other words, the area with $\frac{d\tau}{dt}>0$ is inhabited by particles with positive energy $E=mc^2=\hbar\omega>0$;
2. the area where $\frac{d\tau}{dt}<0$ is inhabited by particles that move into past in respect to the axis of time (reverse flow of time). This is the mirror Universe. Particles that inhabit it are the same as those in our world, but their relativistic masses and frequencies are negative. In other words, the mirror Universe where $\frac{d\tau}{dt}<0$ is the "habitat" of particles with negative energy $E=mc^2=\hbar\omega<0$;
3. the area where $\frac{d\tau}{dt}=0$ is inhabit by a special kind of particles. Their relativistic masses and frequences are zeroes. One may think of such area as a mere surface that separates our world and the mirror Universe. But later we will see that a detailed study has proven: the condition

[5]Hafele J. C. and Keating R. E. Around-the-world atomic clocks: observed relativistic time gains. *Science*, 1972, v. 177, No. 4044, 168–170.

of observable time stop $\frac{d\tau}{dt}=0$ is true in an entire area of space-time, which is characterized by its specific physical properties.

Further, chronometrically invariant (observable) equations of motion for particles with direct flow of time and for those with reverse flow of time are asymmetric, i. e. for particles motion either into past or into future is not the same. This means physical conditions for motion into past or into future are different. And such asymmetry depends upon rotation of space or its deformation.

Evidently, if flow of observable time $d\tau$ was not different from flow of coordinate time dt, the very statement of a problem of areas of space-time with either direct or reverse flow of time would be impossible.

Here we come to a question which I have heard scores of times from my colleagues. Let's assume four independent coordinate axes — one temporal and three spatial. From geometric viewpoint both directions along the time axis are absolutely equal. But what asymmetry are we speaking about and isn't it a sort of mistake?

No, it's not a mistake. Of course, if spatial section (three-dimensional space) is uniform and isotropic, then both directions, into past and into future, are equal. But as soon as the spatial section becomes rotated or deformed (like a twisted sheet of paper set on "the axis of time" and spined around it), the space-time becomes anisotropic in respect to past and future. This anisotropy leads to different physical conditions in motion into past and into future.

As a result, in real space-time we have two different four-dimensional areas — our world with direct flow of time and the mirror Universe with reverse flow of time. These areas are separated with space-time membrane, on which time stops from viewpoint of a regular observer $d\tau=0$, i. e. observer's function of time $\frac{d\tau}{dt}=0$.

What sort of membrane is that and isn't it a mere border surface between our world and the mirror Universe? Our study of the question using method of physical observable values gave the following results.

So, as well-known two types of trajectories are allowable in four-dimensional pseudo-Riemannian space (the basic space-time of General Relativity):

1. non-isotropic trajectories, along which four-dimensional interval is not zero $ds^2=c^2d\tau^2-d\sigma^2\neq0$, three-dimensional observable interval is not zero $d\sigma\neq0$ and interval of observable time is not zero $d\tau\neq0$. These are trajectories of mass-bearing particles that move at sub-light speed (real particles) and at super-light speed (tachyons);
2. isotropic trajectories. These are trajectories of massless light-like particles, which move at the speed of light. Along isotropic trajectories four-dimensional interval is zero $ds^2=0$, but three-dimensional observable interval and interval of observable time are not zeroes $d\sigma^2=c^2d\tau^2\neq0$.

We have also considered trajectories of third type, along which space-time interval is zero $ds^2=c^2d\tau^2-d\sigma^2=0$, three-dimensional interval and interval of observable time are zeroes too $c^2d\tau^2=d\sigma^2=0$. Mathematically that means *full degeneration* of space-time. What is fully degenerated space-time and does it contain any particles? From viewpoint of regular general covariant methods, not related to any frame of reference, here we have absolute zero and the very statement of the problem is senseless. Therefore we called fully degenerated space-time *zero-space*, and the particles which inhabit it — *zero-particles*. But the method of physical observable values, linked to a real frame of reference and its properties, allows an observer to "penetrate" inside zero-space and to see what is going on there.

As a result, we discovered that zero-space contained the whole world with its own coordinates, trajectories and particles. But because of the structure of four-dimensional space-time a regular observer on Earth sees all zero-space shrunk into a single point where observable time stops. But that does not mean that the only way to enter zero-space from our world is through a single special point. To the contrary, entrance is possible at any point. What is necessary is to create physical condition of degeneration in the local space of the entering object. The conditions imply a special combination of gravitational potential w, of space rotation velocity v_i and of the penetrating object's own velocity u^i, which finally takes the form $w+v_iu^i=c^2$. In a partial case, in absence of space's rotation or if the object rests, the condition of degeneration coincides with the condition of collapse $w=c^2$, i. e. entering zero-space is possible also through a black hole.

Because interval of time and spatial interval in zero-space are observed from our world as zeroes, any displacements of zero-particles are instantaneous from the viewpoint of a regular observer. We call such way of interaction *long-range action*. Because particles of our world can not move instantaneously, they can not carry long-range action. But if interaction is transmitted through zero-space (by means of exchange of zero-particles), long-range action becomes possible, because the observed time between emission and reception of signal becomes zero.

Further studies showed that zero-particles also have mass and own frequency, but to see them the observer must enter zero-space themselves. But how do zero-particles look like from the viewpoint of our world's observer? Can we detect them in experiments? We have looked at this problem within de Broglie's particle-wave concept. As a result we have found that for a regular observer zero-particles are *standing waves*.

From geometric viewpoint trajectories in zero-space are the ultimate case of full degeneration of light-like (isotropic) trajectories. Hence zero-particles are the ultimate case of light-like particles, *light-like standing waves*, or in other words, *waves of standing light* (*standing-light hologram*). It is possible that "stop-light experiments" done in Harvard by Lene Hau's group and independently by M. Lukin and R. Walsworth may be an experimental "foreword" to discovery of zero-particles.

Further study of zero-particles showed that their motion breaks the relationship between energy and impulse. Geometrically that means that the square of zero-particle's own vector is not constant in its parallel transfer along its trajectory in zero-space. As known, the square of vector of mass-bearing and light-like particles is conserved along their trajectories. In other words, in "habitats" of regular particles (our world and the mirror Universe) the relationship between energy and impulse is true, but this is not the case in zero-space.

In up-to-date science the one and only type of particles is known for which the relationship between energy and impulse is not true. Those are virtual particles. According to contemporary views based on experimental data, virtual particles carry interaction between any two observable particles (either mass-bearing or light-like). This fact allows unambiguous interpretation of zero-particles and zero-space: (a) zero-particles are virtual particles that carry interaction between any regular particles, and (b) zero-space is an area inhabited by virtual particles, and, at the same time, this is the membrane between our world and the mirror Universe[6].

A natural question arises. If particles of our world interact with each other by means of exchange of zero-particles in zero-space and particles in the mirror Universe do the same, why mixed interactions are never observed, i. e. why interaction between our world and the mirror Universe is never observed?

Of course our every-day experience shows no signs of such interaction. But whether such interaction does not happen at all is a question worth thinking of.

In relation to this problem our colleague B. M. Levin brought our attention to anomalies in annihilation of orthopositronium (λ_{T}-anomaly and isotope anomaly) experimentally discovered in 1987 and still awaiting theoretical explanation. One of these (λ_{T}-anomaly) was discovered in Michigan University[7] (Ann Arbor, USA) and caused the measured life span of orthopositronium to be significantly shorter (by 0.2%) than the theoretical one as predicted by Quantum Electrodynamics up to the accuracy of 0.0007%. Regular annihilation of orthopositronium produces odd number of γ-quanta ($3, 5, 7, \ldots$, largely 3-photon annihilation is observed, because other modes are very small) and is clearly explained by QED. That means that in the experiments mentioned in the above a new factor was observed not explained by QED. The factor appears in β^{+}-decay of nuclei, which was used in these experiments as the source of positrons to produce positronium in substance. Isotope anomaly was experimentally discovered in Gatchina (Russia), and showed collective excitation of ^{22}Ne nuclei that lasted through the entire life of orthopositronium[8]. In 1995 Levin suggested that λ_{T}-anomaly and isotope anomaly were linked to possible 1-photon orthopositronium annihilation, which caused one γ-quantum to be emitted into our world and two γ-quanta (with the total energy of $\sim 3.6 \cdot 10^{-4}$ eV)

[6]For detailed interpretation of virtual particles as zero-particles see Chapter 6 in our *Fields, Vacuum, and the Mirror Universe*.

[7]Westbrook C. I., Gidley D. W., Conti R. S., and Rich A. New precision measurement of the orthopositronium decay rate: a discrepancy with theory. *Physical Review Letters*, 1987, v. 58, 1328–1331.

[8]Levin B. M., Kochenda L. M., Markov A. A., and Shantarovich V. P. Time spectra of annihilation of positrons (^{22}Na) in gaseous neon of various isotopic compositions. *Soviet Journal Nucl. Physics*, 1987, v. 45(6), 1119–1120.

to be emitted into the mirror Universe thus becoming unavailable for observation[9]. Single-photon annihilation of orthopositronium breaks law of conservation of impulse, but taking into account the emission into the mirror Universe (3-photon "mixed" mode) restores the necessary balance.

Possible interactions with the mirror Universe have been discussed since long ago[10], but using methods of quantum theory of fields. No one had approached this problem using geometric methods of General Relativity before we did so.

First we looked at annihilation *per se* as observable interaction between two particles in our world by means of exchange of zero-particles in zero-space. As a result we obtained that two types of zero-particles exist, namely: (a) regular zero-particles, which we called simply *virtual photons*, and (b) zero-particles, i. e. virtual photons in the state of collapse (*virtual collapsers*). Noteworthy, from the viewpoint of an observer on the Earth regular virtual photons carry interaction between particles of our world, while virtual collapsers "are in charge" of interactions in the mirror Universe.

Further we looked at the condition of degeneration of space-time for an electron-positron system, because at the moment of their annihilation interaction is carried by zero-particles (virtual photons)[11].

We found that in annihilation of parapositronium (summary spin is zero) energy of interaction is transmitted by two regular virtual photons, which in our world generate two annihilation γ-quanta. In other words, all products of annihilation are released into our world.

In annihilation of orthopositronium (spin 1) the process is as follows. Non-zero spin causes three photons to carry interaction here. Two "basic" virtual photons are generated as a result of interaction between own energies of electron and positron. The third one results from transformation of spin-energy of orthopositronium, i. e. of additional energy carried by orthopositronium thanks to its non-zero inner momentum (spin). Analysis of the condition of degeneration of space-time shows two possible channels to convey interaction between electron and positron here:

1. interaction is carried by three regular virtual photons. Hence all three γ-quanta are emitted into our world;
2. two "basic" virtual photons collapse to become virtual collapsers. Hence two γ-quanta they generate go to the mirror Universe beyond the reach of an observer. Third zero-particle produced by spin-energy of orthopositronium is a regular virtual photon and generates a γ-quantum that goes to our world. We call this effect "2+1 split of 3-photon annihilation of orthopositronium".

Experiments show that 99.8% of atoms of orthopositronium decay into three γ-quanta as predicted by standard Quantum Electrodynamics. Only 0.2% show either λ_{T}-anomaly (2+1 split of 3-photon annihilation). Hence in annihilation of 99.8% of atoms exchange between electron and positron is effected by regular virtual photons (channel 1). But 0.2% of atoms decay through channel 2, when exchange is effected by 1 regular virtual photon and 2 virtual collapsers. It is these 0.2% of atoms for which the observed effect of "anomalous" 1-photon annihilation is possible, when 1 photon is released into our world and 2 photons are released into the mirror Universe.

To those interested in further information on these and other obtained results I would like to recommend our books *Fields, Vacuum, and the Mirror Universe* and *Particles Here and Beyond the Mirror*. Thank you very much for your kind attention!

May 14, 2001 *D. D. Rabounski*

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[9] Levin B. M. On the kinematics of one-photon annihilation of orthopositronium. *Physics of Atomic Nuclei*, 1995, v. 58(2), 332–334. For this Levin's supposition see also the recent publication: Levin B. M., Borissova L. B., and Rabounski D. D. Orthopositronium and space-time effects. Lomonossov Workshop, Moscow–St. Petersburg, 1999 (*in Russian*).

[10] Holdom B. Two U(1)'s and ϵ charge shifts. *Physics Letters B*, 1986, v. 166, 196–198. Glashow S. L. Positronium versus the mirror Universe. *Physics Letters B*, 1986, v. 167, 35–36.

[11] For detailed theoretical explanation and the history of orthopositronium anomalies see Chapter 6 of *Fields, Vacuum, and the Mirror Universe*.

1 Introduction

The main goal of the theory of motion of particles is to define three-dimensional (spatial) coordinates of a particle at any given moment of time. To attain the goal one should be aware of three things. First, one should know in what sort of space-time the events take place. That is, one should know the geometric structure of space-time, just as one should know the road conditions to drive on it. Second, one should know the physical properties of the moving particle. Third, knowledge of equations of motion of particles of that type is necessary.

The first problem actually leads to choice of a multi-dimensional space from those known in mathematics, which geometry best fits the geometry of the observed world. In the early 20th century Albert Einstein proposed four-dimensional pseudo-Riemannian space with sign-alternating Minkowski signature (one temporal axis and three spatial axes) as the space-time of the observed world. Further development of this assumption led to the General Theory of Relativity, the first geometric theory of space-time and of motion of particles ever since the dawn of the contemporary science.

Successful experiments to verify General Relativity over the recent 80 years explicitely say that four-dimensional pseudo-Riemannian space is the basic space-time of the observed world (as far as the up-to-date measurements' precision allows to judge). And if inevitable evolution of human civilization thought and of experimental technology shows that four-dimensional pseudo-Riemannian space can no-longer explain results of new experiments, this will mean nothing but a more general space should be assumed, which will include four-dimensional pseudo-Riemannian space as a partial case.

This book will focus on motion of particles basing on the geometric concept of the world's structure: we will assume that the geometry of space-time defines all properties of the observed world. Therefore, contrary to other researchers, we are not going to constrain the geometry of space-time by any limitations and we will solve our problems in the way the geometry of space-time requires them to be solved.

Hence, any particle in the space-time corresponds to its own "world" line, which sets three-dimensional (spatial) coordinates of the particle at any given moment of time. Subsequently, our goal to define possible types of particles evolves to considering all allowable types of trajectories of motion in four-dimensional space-time.

Generally, referring to equations of motion of free particles in metric space-time one actually refers to *equations of geodesic lines*, which are four-dimensional equations of trajectories of free particles

$$\frac{d^2x^\alpha}{d\rho^2}+\Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{d\rho}\frac{dx^\nu}{d\rho}=0\,,\qquad \alpha,\mu,\nu=0,1,2,3, \tag{1}$$

where $\Gamma^\alpha_{\mu\nu}$ are Christoffel symbols of 2nd kind and ρ is a parameter of derivation to the geodesic lines. From geometric viewpoint equations of geodesic lines are equations of *parallel transfer in Levi-Civita meaning* [1] of four-dimensional kinematic vector $Q^\alpha=\frac{dx^\alpha}{d\rho}$

$$\frac{DQ^\alpha}{d\rho}=\frac{dQ^\alpha}{d\rho}+\Gamma^\alpha_{\mu\nu}Q^\mu\frac{dx^\nu}{d\rho}=0\,, \tag{2}$$

where D is absolute differential. Kinematic vector Q^α is transferred parallel to itself and tangentially to the trajectory of transfer (a geodesic line). Levi-Civita parallel transfer implies that the length of the transferred vector is conserved

$$Q_\alpha Q^\alpha=g_{\alpha\beta}Q^\alpha Q^\beta=const\,, \tag{3}$$

where $g_{\alpha\beta}$ is fundamental metric tensor.

But the equations of geodesic lines are pure kinematic ones, as they do not contain physical properties of the moving objects. Therefore to obtain the full picture of motion of particles we have to build *dynamic equations of motion*, solving which will give us not trajectories of the particles alone, but their properties (energy, frequency etc.) as well.

To do this we have to define: (1) possible types of trajectories in four-dimensional space-time (pseudo-Riemannian space); (2) dynamical vector for each type of trajectory; (3) derivation parameter to each type of trajectories.

First we consider what types of trajectories are allowable in four-dimensional pseudo-Riemannian space. Along any geodesic line the condition $g_{\alpha\beta}Q^{\alpha}Q^{\beta}{=}const$ is true. If along geodesic lines $g_{\alpha\beta}Q^{\alpha}Q^{\beta}{\neq}0$, such geodesic lines are referred to as *non-isotropic* ones. Along non-isotropic lines the square of four-dimensional interval is not zero

$$ds^2 = g_{\alpha\beta}dx^{\alpha}dx^{\beta} \neq 0\,. \tag{4}$$

and the interval ds becomes

$$ds = \sqrt{g_{\alpha\beta}dx^{\alpha}dx^{\beta}} \qquad \text{if} \quad ds^2 > 0\,, \tag{5}$$

$$ds = \sqrt{-g_{\alpha\beta}dx^{\alpha}dx^{\beta}} \qquad \text{if} \quad ds^2 < 0\,. \tag{6}$$

If along geodesic lines $g_{\alpha\beta}Q^{\alpha}Q^{\beta}{=}0$, such geodesic lines are referred to as *isotropic* ones. Along isotropic geodesic lines the square of four-dimensional interval is zero

$$ds^2 = g_{\alpha\beta}dx^{\alpha}dx^{\beta} = 0\,, \tag{7}$$

but three-dimensional observable interval and interval of observable time are not zeroes.

This ends the list of types of trajectories in four-dimensional pseudo-Riemannian space (the basic space-time of General Relativity).

But other trajectories are theoretically allowable, along which four-dimensional interval, interval of observable time and three-dimensional observable interval are zeroes. Such trajectories lay beyond four-dimensional pseudo-Riemannian space. These are trajectories in fully degenerated space-time. We call it degenerated because from the viewpoint of a regular observer of the Earth all distances and intervals of time in such space degenerate into zero. Nevertheless, transition into such degenerated space from the regular space-time is quite possible (provided certain physical conditions are achieved). And perhaps for the observer, who moves into such degenerated space-time, the terms "time" and "space" will not become void, but will be measured in different units.

Therefore we may consider four-dimensional pseudo-Riemannian space and degenerated space-time as *generalized space-time*, in which both non-degenerated (isotropic and non-isotropic) and degenerated trajectories exist.

Hence in generalized four-dimensional space-time, which is the "extension" of the basic space-time of General Relativity, three types of trajectories are allowable:

1. non-isotropic trajectories (pseudo-Riemannian space). Along them motion is possible at sub-light and super-light speed;
2. isotropic trajectories (pseudo-Riemannian space). Motion along them is possible at light speed only;
3. fully degenerated trajectories (zero-trajectories), which lay in fully degenerated space-time.

According to these three types of trajectories three types of particles can be distinguished, which can exist in four-dimensional space-time:

1. mass-bearing particles (rest-mass $m_0{\neq}0$) move along non-isotropic trajectories ($ds{\neq}0$) at sub-light speed (real mass-bearing particles) and at super-light speed (imaginary mass-bearing particles — tachyons);
2. massless particles (rest-mass $m_0{=}0$) move along isotropic trajectories ($ds{=}0$) at the speed of light. These are light-like particles, e. g. photons;
3. particles of 3rd kind move along trajectories in fully degenerated space-time.

Besides, from pure mathematical viewpoint equations of geodesic lines contain the same vector Q^{α} and the same parameter ρ irrespective of whether the considered trajectories are isotropic or non-isotropic. This hints that there must exist equations of motion, which are the same for mass-bearing and massless particles. We will undertake to search such generalized equations of motion.

In the next Section we will set forth the basics of mathematical apparatus of physical observable values (chronometric invariants), which will serve as our main tool in this book. In Section 3 we will prove existence of generalized dynamic vector and derivation parameter, which are the same for

mass-bearing and massless particles. Section 4 will focus on the conditions for degeneration of pseudo-Riemannian space. Section 5 will consider properties of particles in generalized four-dimensional space-time, which allows degeneration of metric. In Section 6 chronometrically invariant dynamic equations of motion valid for all types of particles will be obtained. In the same Section we will show that Newton's laws of classical mechanics are partial cases of these equations in pseudo-Riemannian space. Section 7 will be devoted to two aspects of the obtained equations: (1) conditions of transformation of generalized space-time into regular space-time, and (2) asymmetry of motion into future (direct flow of time) and into past (reverse flow of time). Sections 8 and 9 will focus on the physical conditions of direct and reverse flow of time. In Section 10 we will look at motion of particles being carried by motion of space itself. Sections from 11 to 13 discuss certain specific conditions (super-light observer, black holes, zero-space). Finally, in Section 14 we will build the general picture of motion of particles in generalized space-time, basing on the results obtained in this book.

2 Physical observable values

To build a descriptive picture of any physical theory we need to express the results through real physical values, which can be measured in experiments (*physical observable values*). In General Relativity this problem is not a trivial one, because we are looking at objects in four-dimensional space-time and we have to define, which components of four-dimensional tensor values are physical observable values.

A mathematical apparatus to calculate physical observable values in four-dimensional pseudo-Riemannian space was first introduced by A. L. Zelmanov and is referred to as *theory of chronometric invariants*. The apparatus was published in 1944 in Zelmanov's doctorate thesis [2] and in his later study of 1956 [3].

Similar results were obtained by C. Cattaneo independently from Zelmanov. Cattaneo published his first study in 1958 [4, 5, 6, 7].

The essence of Zelmanov's apparatus of physical observable values designed for the four-dimensional curved unhomogeneous space-time (pseudo-Riemannian space), is as follows.

In any point of space-time we can place a local *spatial section* $x^0 = ct = const$ (local space) orthogonal to *time line* $x^i = const$. If the spatial section is everywhere orthogonal to time lines, the space is referred to as *holonomic*. Otherwise the space is referred to as *non-holonomic*.

Frames of reference of a real observer include coordinate nets spanned over a real reference body and real clock which can represent a set of real references to which the observer refer his observations. Therefore physical observable values should be a result of projection of four-dimensional values on space and time of the reference body of the observer. Operator of projection on time is the vector of four-dimensional velocity

$$b^\alpha = \frac{dx^\alpha}{ds}, \qquad b_\alpha b^\alpha = +1\,, \tag{8}$$

of the reference body in respect to the observer. Operator of projection on spatial section of the observer (his local space) is defined as four-dimensional symmetric tensor $h_{\alpha\beta}$

$$\begin{aligned} h_{\alpha\beta} &= -g_{\alpha\beta} + b_\alpha b_\beta\,, & h^{\alpha\beta} &= -g^{\alpha\beta} + b^\alpha b^\beta, \\ h_\alpha^\beta &= -g_\alpha^\beta + b_\alpha b^\beta, & h_i^\alpha h_\alpha^k &= \delta_i^k\,, \quad h_\alpha^i b^\alpha = 0\,. \end{aligned} \tag{9}$$

If observer rests in respect to his references (*accompanying frame of reference*, $b^i = 0$) transformation of coordinates only means transition from one coordinate net to another within the same spatial section. Therefore physical observable values in accompanying frames of reference should be invariant in respect to transformation of time, i. e. should be *chronometrically invariant values*. Zelmanov developed the method to calculate chronometrically invariant projections of any four-dimensional value and set it forth as a theorem.

Zelmanov theorem "We assume that $Q_{00\ldots0}^{ik\ldots p}$ are components of four-dimensional tensor $Q_{00\ldots0}^{\mu\nu\ldots\rho}$ of r-th rank, in which all upper indices are not zero, while all m lower indices are zeroes. Then tensor values

$$T^{ik\ldots p} = (g_{00})^{-\frac{m}{2}}\, Q_{00\ldots0}^{ik\ldots p} \tag{10}$$

make up chronometrically invariant three-dimensional contravariant tensor of $(r-m)$-th rank. Hence tensor $T^{ik\dots p}$ is a result of m-fold projection on time by indices $\alpha, \beta \dots \sigma$ and projection on space by $r-m$ indices $\mu, \nu \dots \rho$ of the initial tensor $Q^{\mu\nu\dots\rho}_{\alpha\beta\dots\sigma}$".

According to Zelmanov theorem chronometrically invariant (physical observable) projections of four-dimensional vector Q^α are values

$$b^\alpha Q_\alpha = \frac{Q_0}{\sqrt{g_{00}}}, \qquad h^i_\alpha Q^\alpha = Q^i. \tag{11}$$

In accordance to this theorem physical observable projections of symmetric tensor of the 2nd rank $Q^{\alpha\beta}$ are values

$$b^\alpha b^\beta Q_{\alpha\beta} = \frac{Q_{00}}{g_{00}}, \quad h^{i\alpha} b^\beta Q_{\alpha\beta} = \frac{Q^i_0}{\sqrt{g_{00}}}, \quad h^i_\alpha h^k_\beta Q^{\alpha\beta} = Q^{ik}. \tag{12}$$

Projections of four-dimensional coordinate interval dx^α are interval of physical observable time

$$d\tau = \sqrt{g_{00}}\, dt + \frac{g_{0i}}{c\sqrt{g_{00}}} dx^i, \tag{13}$$

and interval of observable coordinates dx^i which are the same as spatial coordinates. Physical observable velocity of a particle is three-dimensional chronometrically invariant vector

$$\mathrm{v}^i = \frac{dx^i}{d\tau}. \tag{14}$$

Projecting covariant or conravariant fundamental metric tensor on spatial section in an accompanying frame of reference

$$\begin{aligned} h^\alpha_i h^\beta_k g_{\alpha\beta} &= g_{ik} - b_i b_k = -h_{ik}\,, \\ h^i_\alpha h^k_\beta g^{\alpha\beta} &= g^{ik} - b^i b^k = g^{ik} = -h^{ik}, \end{aligned} \tag{15}$$

we obtain that $h_{ik} = -g_{ik} + b_i b_k$ is *observable metric tensor* using which we can lift and lower indices of three-dimensional values in accompanying frame of reference. Thus four-dimensional interval and three-dimensional observable one, represented through observable values, are

$$ds^2 = c^2 d\tau^2 - d\sigma^2, \qquad d\sigma^2 = h_{ik} dx^i dx^k. \tag{16}$$

The main physical observable properties of the space of reference were deduced by Zelmanov in his dissertation thesis [2] from the property of non-commutativity (non zero difference between mixed 2nd derivatives with respect to time and spatial coordinates) of his chronometrically invariant operators of derivation $\frac{^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}$ and $\frac{^*\partial}{\partial x^i} = \frac{\partial}{\partial x^i} - \frac{g_{0i}}{g_{00}}\frac{\partial}{\partial x^0}$

$$\frac{^*\partial^2}{\partial x^i \partial t} - \frac{^*\partial^2}{\partial t \partial x^i} = \frac{1}{c^2} F_i \frac{^*\partial}{\partial t}\,, \qquad \frac{^*\partial^2}{\partial x^i \partial x^k} - \frac{^*\partial^2}{\partial x^k \partial x^i} = \frac{2}{c^2} A_{ik} \frac{^*\partial}{\partial t}\,. \tag{17}$$

These properties are characterized by three-dimensional chronometrically invariant tensors: antisymmetric tensor of *angular velocity of rotation of reference's space* A_{ik} and vector of *gravitational inertial force* F_i

$$A_{ik} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x^i} - \frac{\partial v_i}{\partial x^k}\right) + \frac{1}{2c^2}\left(F_i v_k - F_k v_i\right), \tag{18}$$

$$F_i = \frac{c^2}{c^2 - w}\left(\frac{\partial w}{\partial x^i} - \frac{\partial v_i}{\partial t}\right). \tag{19}$$

Here w and v_i are not chronometrically invariant values that characterize body of reference and its reference's space. These are gravitational potential $w = c^2(1 - \sqrt{g_{00}})$ and linear velocity of space's rotation $v_i = -c\frac{g_{0i}}{\sqrt{g_{00}}}$.

The necessary and sufficient condition of holonomity of the space should be equality to zero of the tensor A_{ik}. Naturally, if spatial sections everywhere are orthogonal to time lines in some fixed frame of

reference (holonomic space), then in the frame of reference the values g_{0i} are zeroes. Because g_{0i}=0, we have v_i=0 and A_{ik}=0 too. Therefore the tensor A_{ik} also we will refer to as *tensor of non-holonomity of space*.

In quasi-Newtonian approximation, i. e. in a weak gravitational field at speeds much lower than the speed of light and in absence of rotation of space F_i becomes a regular non-relativistic gravitational force $F_i = \frac{\partial w}{\partial x^i}$.

In addition, the reference body can be deformed which should be also taken into account in measurements. This can be done introducing into the equations a three-dimensional symmetric chronometrically invariant tensor of *deformation velocities* of the space of reference

$$D_{ik} = \frac{1}{2}\frac{{}^*\partial h_{ik}}{\partial t}\,, \qquad D^{ik} = -\frac{1}{2}\frac{{}^*\partial h^{ik}}{\partial t}\,,$$
$$D = D^k_k = \frac{{}^*\partial \ln\sqrt{h}}{\partial t}\,, \qquad h = \det\|h_{ik}\|\;. \tag{20}$$

Given these definitions we can generally formulate any geometric object in Riemannian space with observable parameters of the space of reference. For instance, having any equations obtained using general covariant methods we can calculate their chronometrically invariant projections on time and on spatial section of any particular body of reference and formulate them with its real physical observable properties. From here we arrive to equations containing only values measurable in practice.

These are the basics of the mathematical apparatus of physical observable values — Zelmanov's chronometric invariants [2, 3, 8, 9, 10].

3 Motion of mass-bearing and massless particles

According to up-to-date physical concepts [10] mass-bearing particles are characterized by four-dimensional vector of impulse P^α, while massless particles are characterized by four-dimensional wave vector K^α

$$P^\alpha = m_0\frac{dx^\alpha}{ds}\,, \qquad K^\alpha = \frac{\omega}{c}\frac{dx^\alpha}{d\sigma}\,, \tag{21}$$

where ω is frequency that characterizes massless particle. In this case space-time interval ds is taken as the derivation parameter for mass-bearing particles (non-isotropic trajectories, $ds\neq0$). Along isotropic trajectories ds=0 (massless particles), but three-dimensional observable interval $d\sigma\neq0$. Therefore $d\sigma$ is taken as the derivation parameter for massless particles.

The square of impulse vector P^α along trajectories of mass-bearing particles is not zero

$$P_\alpha P^\alpha = g_{\alpha\beta}P^\alpha P^\beta = m_0^2 = const \neq 0\,, \tag{22}$$

i. e. P^α is a non-isotropic vector. The square of wave vector K^α along trajectories of massless particles is zero, i. e. K^α is an isotropic vector

$$K_\alpha K^\alpha = g_{\alpha\beta}K^\alpha K^\beta = \frac{\omega^2}{c^2}\frac{g_{\alpha\beta}dx^\alpha dx^\beta}{d\sigma^2} = \frac{\omega^2}{c^2}\frac{ds^2}{d\sigma^2} = 0\,. \tag{23}$$

Because ds^2 in chronometrically invariant form (16) is

$$ds^2 = c^2d\tau^2 - d\sigma^2 = c^2d\tau^2\left(1-\frac{\mathrm{v}^2}{c^2}\right), \qquad \mathrm{v}^2 = h_{ik}\mathrm{v}^i\mathrm{v}^k, \tag{24}$$

we can put P^α and K^α down as

$$P^\alpha = m_0\frac{dx^\alpha}{ds} = \frac{m}{c}\frac{dx^\alpha}{d\tau}\,, \qquad K^\alpha = \frac{\omega}{c}\frac{dx^\alpha}{d\sigma} = \frac{k}{c}\frac{dx^\alpha}{d\tau}\,, \tag{25}$$

where $k=\frac{\omega}{c}$ is the wave number and m is the relativistic mass. Out of the obtained formulas we can see that physical observable time τ can be used as a universal derivation parameter to both isotropic and non-isotropic trajectories, i. e. as the single derivation parameter for mass-bearing and massless particles.

Calculation of contravariant components of vectors P^{α} and K^{α} gives

$$P^{0}=m\frac{dt}{d\tau}\,,\qquad P^{i}=\frac{m}{c}\frac{dx^{i}}{d\tau}=\frac{1}{c}m\mathrm{v}^{i}\,,\tag{26}$$

$$K^{0}=k\frac{dt}{d\tau}\,,\qquad K^{i}=\frac{k}{c}\frac{dx^{i}}{d\tau}=\frac{1}{c}k\mathrm{v}^{i}\,,\tag{27}$$

where $m\mathrm{v}^{i}$ is three-dimensional vector of impulse of mass-bearing particle and $k\mathrm{v}^{i}$ is three-dimensional wave vector of massless particle.

The formula for $\frac{dt}{d\tau}$ can be obtained from the square of vector of four-dimensional velocity of particle U^{α}, which for sub-light speed, light speed and super-light speed is, respectively

$$g_{\alpha\beta}U^{\alpha}U^{\beta}=+1\,,\qquad U^{\alpha}=\frac{dx^{\alpha}}{ds}\qquad ds=cd\tau\sqrt{1-\frac{\mathrm{v}^{2}}{c^{2}}}\,,\tag{28}$$

$$g_{\alpha\beta}U^{\alpha}U^{\beta}=0\,,\qquad U^{\alpha}=\frac{dx^{\alpha}}{d\sigma}\qquad ds=0\,,\qquad d\sigma=cd\tau\,,\tag{29}$$

$$g_{\alpha\beta}U^{\alpha}U^{\beta}=-1\,,\qquad U^{\alpha}=\frac{dx^{\alpha}}{|ds|}\qquad ds=cd\tau\sqrt{\frac{\mathrm{v}^{2}}{c^{2}}-1}\,.\tag{30}$$

Substituting the definitions for h_{ik}, v_i and v^{i} into each formula for $g_{\alpha\beta}U^{\alpha}U^{\beta}$ we arrive to three quadratic equations in respect to $\frac{dt}{d\tau}$. They are the same for sub-light, light-like and super-light speeds

$$\left(\frac{dt}{d\tau}\right)^{2}-\frac{2v_i\mathrm{v}^{i}}{c^{2}\left(1-\frac{w}{c^{2}}\right)}\frac{dt}{d\tau}+\frac{1}{\left(1-\frac{w}{c^{2}}\right)^{2}}\left(\frac{1}{c^{4}}v_iv_k\mathrm{v}^{i}\mathrm{v}^{k}-1\right)=0\,.\tag{31}$$

This quadratic equation has two solutions

$$\left(\frac{dt}{d\tau}\right)_{1,2}=\frac{1}{1-\frac{w}{c^{2}}}\left(\frac{1}{c^{2}}v_i\mathrm{v}^{i}\pm 1\right).\tag{32}$$

Function $\frac{dt}{d\tau}$ allows to define what direction in time the particle takes. If $\frac{dt}{d\tau}{>}0$ then temporal coordinate t increases, i. e. the particle moves from past into future (direct flow of time). If $\frac{dt}{d\tau}{<}0$ then temporal coordinate decreases, i. e. the particle moves from future into past (reverse flow of time).

The value $1-\frac{w}{c^{2}}=\sqrt{g_{00}}>0$, because other cases $\sqrt{g_{00}}=0$ and $\sqrt{g_{00}}<0$ contradict the signature conditions $(+---)$. Therefore the coordinate time t stops $\frac{dt}{d\tau}=0$ provided

$$v_i\mathrm{v}^{i}=-c^{2},\qquad\quad v_i\mathrm{v}^{i}=+c^{2}.\tag{33}$$

The coordinate time t has direct flow $\frac{dt}{d\tau}{>}0$ if in the first and in the second solutions, respectively

$$\frac{1}{c^{2}}v_i\mathrm{v}^{i}+1>0\,,\qquad\quad \frac{1}{c^{2}}v_i\mathrm{v}^{i}-1>0\,.\tag{34}$$

The coordinate time t has reverse flow $\frac{dt}{d\tau}{<}0$ at

$$\frac{1}{c^{2}}v_i\mathrm{v}^{i}+1<0\,,\qquad\quad \frac{1}{c^{2}}v_i\mathrm{v}^{i}-1<0\,.\tag{35}$$

For sub-light speed particles $v_i\mathrm{v}^{i}<c^{2}$ is always true. Hence direct flow of time for regularly observed mass-bearing particles takes place under the first condition from (34) while reverse flow of time takes place under the second condition from (35).

Noteworthy, we looked at the problem of the direction of coordinate time t assuming that physical observable time is $d\tau>0$ always.

Now using formulas (26), (27), and (32) we calculate covariant components P_i and K_i as well as projections of four-dimensional vectors P^{α} and K^{α} onto time

$$P_i=-\frac{m}{c}\left(\mathrm{v}_i\pm v_i\right),\qquad K_i=-\frac{k}{c}\left(\mathrm{v}_i\pm v_i\right),\tag{36}$$

$$\frac{P_0}{\sqrt{g_{00}}} = \pm m\,, \qquad \frac{P_0}{\sqrt{g_{00}}} = \pm k\,, \tag{37}$$

where values $+m$ and $+k$ take place in observation of particles that move into future (direct flow of time), while values $-m$ and $-k$ take place in observation of particles that move into past (reverse flow of time).

Therefore, physical observable values are as follows. For mass-bearing particles these are relativistic mass $\pm m$ and three-dimensional value $\frac{1}{c}m\mathrm{v}^i$, where $m\mathrm{v}^i$ is observable vector of impulse. For massless particles these are wave number $\pm k$ and three-dimensional value $\frac{1}{c}k\mathrm{v}^i$, where $k\mathrm{v}^i$ is observable wave vector.

From the obtained formulas (36) and (37) we can see that observable wave vector of massless particles $k\mathrm{v}^i$ is a full analog to observable vector of impulse of mass-bearing particles $m\mathrm{v}^i$.

Substituting the obtained values P^0, P^i, K^0, K^i, and g_{ik}, expressed through $h_{ik}{=}-g_{ik}{+}\frac{1}{c^2}v_i v_k$ into the formulas for $P_\alpha P^\alpha$ (22) and $K_\alpha K^\alpha$ (23) we arrive to the relationships between physical observable energy and physical observable impulse for mass-bearing particle

$$\frac{E^2}{c^2} - m^2\mathrm{v}_i\mathrm{v}^i = \frac{E_0^2}{c^2}\,, \tag{38}$$

and for massless particle

$$h_{ik}\mathrm{v}^i\mathrm{v}^k = c^2, \tag{39}$$

where $E{=}\pm mc^2$ is relativistic energy of the particle, and $E_0{=}m_0c^2$ is its rest-energy.

Therefore, by comparing the notations for P^α and K^α we obtained the universal derivation parameter τ, which is the same for both mass-bearing and massless particles. But four-dimensional dynamic vectors P^α and K^α themselves *are different*.

Now we are going to calculate the universal dynamic vector, which partial cases are dynamic vector of mass-bearing particles P^α and dynamic vector of massless particles K^α.

We will tackle the problem assuming that the wave-particle dualism is peculiar to all particles without any exception. That is, we will consider motion of massless and mass-bearing particles as propagation of waves in geometric optics approximation. Four-dimensional wave vector of massless particles K^α in geometric optics approximation is [11]

$$K_\alpha = \frac{\partial\psi}{\partial x^\alpha}\,, \tag{40}$$

where ψ is wave phase (eikonal). In a similar way we put down the four-dimensional vector of impulse of mass-bearing particles P^α

$$P_\alpha = \frac{\hbar}{c}\frac{\partial\psi}{\partial x^\alpha}\,, \tag{41}$$

where $\hbar$ is Planck constant, and coefficient $\frac{\hbar}{c}$ equalizes the dimensions of both parts of the equation. From these formulas we arrive to

$$\frac{K_0}{\sqrt{g_{00}}} = \frac{1}{c}\frac{^*\partial\psi}{\partial t}\,, \qquad \frac{P_0}{\sqrt{g_{00}}} = \frac{\hbar}{c^2}\frac{^*\partial\psi}{\partial t}\,. \tag{42}$$

Equalizing the values (42) to (37) we obtain

$$\pm\,\omega = \frac{^*\partial\psi}{\partial t}\,, \qquad \pm m = \frac{\hbar}{c^2}\frac{^*\partial\psi}{\partial t}\,. \tag{43}$$

From here we see that the values $+\omega$ for massless particles and $+m$ for mass-bearing particles take place at wave phase ψ increasing with time, while $-\omega$ and $-m$ take place at wave phase decreasing with time. From these expressions we obtain the relationship for particle's energy, which takes into account its dual (wave-particle) nature

$$\pm\, mc^2 = \pm\,\hbar\omega = \hbar\frac{^*\partial\psi}{\partial t} = E\,. \tag{44}$$

Now from formula (41) we obtain the dependence between chronometrically invariant impulse of particle p^i and its phase ψ

$$p^i = m\mathrm{v}^i = -\hbar h^{ik}\frac{^*\partial\psi}{\partial x^k}\,, \qquad p_i = m\mathrm{v}_i = -\hbar\frac{^*\partial\psi}{\partial x^i}\,. \tag{45}$$

Further, the condition $K_\alpha K^\alpha$ can be presented in the form of [11]

$$g^{\alpha\beta}\frac{\partial\psi}{\partial x^\alpha}\frac{\partial\psi}{\partial x^\beta} = 0\,, \tag{46}$$

which is the basic equation of the geometric optics (eikonal equation). Formulating regular operators of derivation with chronometrically invariant operators $\frac{^*\partial}{\partial t}$ and $\frac{^*\partial}{\partial x^i}$ and taking into account that

$$g^{00} = \frac{1 - \dfrac{1}{c^2}v_i v^i}{g_{00}}, \quad g^{ik} = -h^{ik}, \quad v^i = h^{ik}v_k = -cg^{0i}\sqrt{g_{00}}\,, \tag{47}$$

we immediately arrive to eikonal equation for massless particles in chronometrically invariant form

$$\frac{1}{c^2}\left(\frac{^*\partial\psi}{\partial t}\right)^2 + h^{ik}\frac{^*\partial\psi}{\partial x^i}\frac{^*\partial\psi}{\partial x^k} = 0\,. \tag{48}$$

In a similar way we obtain chronometrically invariant eikonal equation for mass-bearing particles

$$\frac{1}{c^2}\left(\frac{^*\partial\psi}{\partial t}\right)^2 + h^{ik}\frac{^*\partial\psi}{\partial x^i}\frac{^*\partial\psi}{\partial x^k} = \frac{m_0^2 c^2}{\hbar^2}\,, \tag{49}$$

which at m_0=0 becomes the same as the former one.

Substituting relativistic mass m (43) into (25) we obtain the dynamic vector P^α that describes motion of massless and mass-bearing particles in the geometric optics approximation

$$P^\alpha = \frac{\hbar\omega}{c^3}\frac{dx^\alpha}{\partial\tau}\,, \qquad P_\alpha P^\alpha = \frac{\hbar^2\omega^2}{c^4}\left(1 - \frac{\mathrm{v}^2}{c^2}\right). \tag{50}$$

The length of the vector is a real value at $\mathrm{v}<c$, is zero at $\mathrm{v}=c$ and is an imaginary value at $\mathrm{v}>c$. Therefore the obtained dynamic vector P^α characterizes motion of particles with any rest-mass (real, zero or imaginary).

Observable projections of the obtained universal vector P^α are

$$\frac{P_0}{\sqrt{g_{00}}} = \pm\frac{\hbar\omega}{c^2}\,, \qquad P^i = \frac{\hbar\omega}{c^3}\mathrm{v}^i, \tag{51}$$

where the time observable projection has the mass dimension and the value p^i=cP^i has the dimension of impulse.

4 Degenerated space-time. Zero-particles

As known, along trajectories of massless particles (isotropic trajectories) the square of interval of four-dimensional wave is zero

$$ds^2 = c^2d\tau^2 - d\sigma^2 = 0\,, \qquad c^2d\tau^2 = d\sigma^2 = 0\,. \tag{52}$$

But ds^2=0 not only at $c^2d\tau^2$=$d\sigma^2$, but also when even stricter condition is true, $c^2d\tau^2$=$d\sigma^2$=0. The condition $d\tau^2$=0 means that physical observable time τ has the same value along the entire trajectory. The condition $d\sigma^2$=0 means that all three-dimensional trajectories have zero lengths. Taking into account the definitions of $d\tau$ (13), $d\sigma^2$ (16) and the fact that in an accompanying system of reference h_{00}=h_{0i}=0, we put down the conditions $d\tau^2$=0 and $d\sigma^2$=0 as

$$cd\tau = \left[1 - \frac{1}{c^2}\left(w + v_i u^i\right)\right]cdt = 0\,, \qquad dt \neq 0\,, \tag{53}$$

$$d\sigma^2 = h_{ik}dx^i dx^k = 0\,, \tag{54}$$

where $u^i{=}\frac{dx^i}{dt}$ is three-dimensional coordinate velocity of particle, which is not a chronometrically invariant (physical observable) value.

As known, the necessary and sufficient condition of full degeneration of quadratic metric is equality to zero of the determinant of its metric tensor. For a three-dimensional physical observable metric $d\sigma^2{=}h_{ik}dx^i dx^k$ this condition is $h{=}\det||h_{ik}||{=}0$. But the determinant of the observable three-dimensional metric tensor h_{ik} has the form [10]

$$h = -\frac{g}{g_{00}}\,, \qquad g = \det||g_{\alpha\beta}||\,. \tag{55}$$

Hence if three-dimensional form $d\sigma^2$ is degenerated $h{=}0$, then four-dimensional form ds^2 is also degenerated $g{=}0$. Hence four-dimensional space-time, where conditions (53) and (54) are true, is *a fully degenerated space-time*.

Substituting $h_{ik}{=}-g_{ik}{+}\frac{1}{c^2}v_i v_k$ into (54) and dividing it by dt^2 we obtain *physical conditions of degeneration* (53) and (54) in the final form

$$w + v_i u^i = c^2, \qquad g_{ik}u^i u^k = c^2\left(1-\frac{w}{c^2}\right)^2, \tag{56}$$

where $v_i u^i$ is the scalar product of space's rotation velocity v_i and coordinate velocity of particle u^i.

If all values $v_i{=}0$ (holonomic space), then $w{=}c^2$ and $\sqrt{g_{00}}{=}1{-}\frac{w}{c^2}{=}0$. That means that gravitational potential of the body of reference w is strong enough to bring the body of reference to gravitational collapse (*black hole*). This case is not discussed here.

In the below we are going to look at degeneration of four-dimensional space-time, when three-dimensional space is non-holonomic $v_i{\neq}0$, i. e. rotates.

Using the definition of interval of observable time $d\tau$ (13), we obtain the relationship between coordinate velocity of particle u^i and its observable velocity v^i

$$\mathrm{v}^i = \frac{u^i}{1-\frac{1}{c^2}\left(w+v_k u^k\right)}\,. \tag{57}$$

Now we can put down ds^2 in a form to have the conditions of degeneration presented explicitly

$$ds^2 = c^2 d\tau^2\left(1-\frac{\mathrm{v}^2}{c^2}\right) = c^2 dt^2\left\{\left[1-\frac{1}{c^2}\left(w+v_k u^k\right)\right]^2 - \frac{u^2}{c^2}\right\}. \tag{58}$$

Evidently degenerated space-time can only host the particles for which physical conditions of degeneration (56) are true.

We will refer to the particles that move in fully degenerated space-time (*zero-space*) as *zero-particles.*

5 Generalized space-time for particles of three kinds

Looking at motion of mass-bearing and massless particles we considered a four-dimensional space-time with strictly non-degenerated metric $g{<}0$. Now we are going to consider a four-dimensional space-time where degeneration of metric $g{\leq}0$ is possible.

We already obtained the metric of such generalized space-time in the previous Section (58). Hence vector of impulse of mass-bearing particle P^α in generalized space-time ($g{\leq}0$) takes the form

$$P^\alpha = m_0\frac{dx^\alpha}{ds} = \frac{M}{c}\frac{dx^\alpha}{dt}\,, \tag{59}$$

$$M = \frac{m_0}{\sqrt{\left[1-\frac{1}{c^2}\left(w+v_k u^k\right)\right]^2 - \frac{u^2}{c^2}}}\,, \tag{60}$$

where M stands for *gravitational rotational mass* of particle. Gravitational rotational mass M depends not only upon three-dimensional velocity of particle, but upon gravitational potential w (field of the body of reference) and upon velocity of rotation v_i of the space itself.

From the obtained formula (59) we see that in four-dimensional space-time where degeneration of metric is possible ($g \leq 0$), the generalized parameter of derivation is coordinate time t.

Substituting v^2 from (57) and $m_0 = m\sqrt{1-\mathrm{v}^2/c^2}$ into this formula, we arrive to the relationship between relativistic mass of particle m and its gravitational rotational mass M

$$M = \frac{m}{1 - \frac{1}{c^2}\left(w + v_i u^i\right)}\,. \tag{61}$$

From the obtained formula we see that M is a ratio between two values, each one equal to zero in case of degenerated metric ($g=0$), but the ratio is not zero $M \neq 0$.

The fact is no surprise. The same is true for relativistic mass m in case of $\mathrm{v}^2 = c^2$, when $m_0 = 0$ and $\sqrt{1-\mathrm{v}^2/c^2} = 0$, but their ratio $m \neq 0$.

Therefore light-like (massless) particles are the ultimate case of mass-bearing ones at $\mathrm{v} \to c$. Zero-particles can be regarded the ultimate case of light-like ones that move in non-holonomic space at observable velocity v^i (57), which depends upon gravitational potential of the body of reference w and upon the direction in respect to the velocity of rotation of space.

The temporal component of vector P^α (59) and its physical observable projection onto time are

$$P^0 = M = \frac{m}{1 - \frac{1}{c^2}\left(w + v_i u^i\right)}\,, \tag{62}$$

$$\frac{P_0}{\sqrt{g_{00}}} = M\left[1 - \frac{1}{c^2}\left(w + v_i u^i\right)\right] = m\,, \tag{63}$$

while its spatial components are

$$P^i = \frac{M}{c}u^i = \frac{m}{c}\mathrm{v}^i, \tag{64}$$

$$P_i = -\frac{M}{c}\left[u_i + v_i - \frac{1}{c^2}v_i\left(w + v_k u^k\right)\right]. \tag{65}$$

Evidently in case of degeneration of space-time, i. e. under physical conditions of degeneration (56), these components become

$$P^0 = M\,, \qquad \frac{P_0}{\sqrt{g_{00}}} = m = 0\,, \tag{66}$$

$$P^i = \frac{M}{c}u^i, \qquad P_i = -\frac{M}{c}u_i\,, \tag{67}$$

i. e. particles that move in degenerated space-time (zero-particles) bear zero relativistic mass, but their gravitational rotational mass $M \neq 0$.

Now we are going to look at motion of mass-bearing particles in generalized space-time within the dual wave-particle concept. Components of the universal dynamic vector $P_\alpha = \frac{\hbar}{c}\frac{{}^*\partial\psi}{\partial x^\alpha}$ (41) here are

$$\frac{P_0}{\sqrt{g_{00}}} = m = M\left[1 - \frac{1}{c^2}\left(w + v_i u^i\right)\right] = \frac{\hbar}{c^2}\frac{{}^*\partial\psi}{\partial t}\,, \tag{68}$$

$$P_i = \frac{\hbar}{c}\left(\frac{{}^*\partial\psi}{\partial x^i} - \frac{1}{c^2}v_i\frac{{}^*\partial\psi}{\partial t}\right), \tag{69}$$

$$P^i = \frac{m}{c}\mathrm{v}^i = \frac{M}{c}u^i = -\frac{\hbar}{c}h^{ik}\frac{{}^*\partial\psi}{\partial x^k}\,, \tag{70}$$

$$P^0 = M = \frac{\hbar}{c^2\left(1-\frac{w}{c^2}\right)}\left(\frac{{}^*\partial\psi}{\partial t} - v^i\frac{{}^*\partial\psi}{\partial x^i}\right). \tag{71}$$

Out of these components the following two formulas can be obtained. First one (72) links gravitational rotational mass M to its corresponding total energy E_{total}. Second one (73) links three-dimensional generalized impulse Mu^i to change of wave phase ψ

$$Mc^2 = \frac{1}{1-\frac{1}{c^2}\left(w+v_iu^i\right)}\,\hbar\frac{{}^*\partial\psi}{\partial t} = \hbar\Omega = E_{total}\,, \tag{72}$$

$$Mu^i = -\,\hbar h^{ik}\frac{{}^*\partial\psi}{\partial x^k}\,, \tag{73}$$

where Ω is gravitational rotational frequency, and ω is regular frequency

$$\Omega = \frac{\omega}{1-\frac{1}{c^2}\left(w+v_iu^i\right)}\,, \qquad \omega = \frac{{}^*\partial\psi}{\partial t}\,. \tag{74}$$

The condition $P_\alpha P^\alpha{=}const$ in approximation of geometric optics (eikonal equation) takes the form (49). For corpuscular form of this condition in generalized space-time we obtain a chronometrically invariant formula

$$\frac{E^2}{c^2} - M^2u^2 = \frac{E_0^2}{c^2}\,, \tag{75}$$

where M^2u^2 is the square of generalized three-dimensional impulse vector, $E{=}mc^2$, and $E_0{=}m_0c^2$. Using this formula we put down the formula for universal dynamic vector P^α, which will include the degeneration conditions

$$P^\alpha = \frac{\hbar\Omega}{c^3}\frac{dx^\alpha}{dt} = \frac{\hbar\frac{{}^*\partial\psi}{\partial t}}{c^3\left[1-\frac{1}{c^2}\left(w+v_iu^i\right)\right]}\frac{dx^\alpha}{\partial t}\,, \tag{76}$$

$$P_\alpha P^\alpha = \frac{\hbar^2\Omega^2}{c^4}\left\{\left[1-\frac{1}{c^2}\left(w+v_iu^i\right)\right]^2 - \frac{u^2}{c^2}\right\}. \tag{77}$$

For degeneration of space-time we have $m{=}0$, $\omega{=}\frac{{}^*\partial\psi}{\partial t}{=}0$, and $P_\alpha P^\alpha{=}0$, i. e. particles that move in degenerated space-time from viewpoint of a regular observer bear zero rest-mass, zero relativistic mass m and zero relativistic frequency ω, which corresponds to relativistic mass within the wave-particle concept. Also within this viewpoint the square of four-dimensional vector (dynamic vector of zero-particles) does conserve $P_\alpha P^\alpha{=}0$. Therefore we call such particles *zero-particles*. For zero-particles gravitational rotational mass M (60), generalized three-dimensional impulse Mu^i (73) and gravitational rotational frequency Ω (74), which corresponds to mass M within the wave-particle concept, are *not zeroes*.

Zero-space's metric $d\mu^2$ is not invariant from viewpoint of inner observer who inhabits zero-space. It can be proven from 2nd condition of degeneration $d\sigma^2{=}h_{ik}dx^idx^k$. Substituting here $h_{ik}{=}{-}g_{ik}{+}\frac{1}{c^2}v_iv_k$, dividing by dt^2, and substituting 1st condition of degeneration $w{+}v_iu^i{=}c^2$ we arrive to inner zero-space's metric

$$d\mu^2 = g_{ik}dx^idx^k = \left(1-\frac{w}{c^2}\right)^2c^2dt^2 \neq inv\,, \tag{78}$$

which is not invariant. Hence, from viewpoint of an observer within zero-space the square of four-dimensional vector of zero-particles does not conserve

$$U_\alpha U^\alpha = g_{ik}u^iu^k = \left(1-\frac{w}{c^2}\right)^2c^2 \neq const. \tag{79}$$

Equation of eikonal (wave phase) for zero-particles can be obtained by substituting the conditions $m{=}0$, $\omega{=}\frac{{}^*\partial\psi}{\partial t}{=}0$, and $P_\alpha P^\alpha{=}0$ into the eikonal equation (48) or (49). As a result we obtain that the eikonal equation for zero-particles from viewpoint of a regular observer is

$$h^{ik}\frac{{}^*\partial\psi}{\partial x^i}\frac{{}^*\partial\psi}{\partial x^k}=0\,, \tag{80}$$

and is a standing wave equation (*information circle*).

As a result we obtain that in non-holonomic space-time two ultimate transitions are possible: (a) *light barrier*, to overcome which a particle should exceed the speed of light, and (b) *zero-transition* for which a particle should be in a state of a specific rotation depending upon particular distribution of matter (conditions of degeneration).

6 Equations of motion

6.1 General considerations

Now we are going to obtain dynamic equations of motion of free particles in generalized space-time $g{\leq}0$, i. e. the common equations of motion for mass-bearing, massless and zero particles.

From geometric viewpoint the equations in question are those of parallel transfer in the meaning of Levi-Civita of a universal dynamic vector P^α

$$DP^\alpha=dP^\alpha+\Gamma^\alpha_{\mu\nu}P^\mu dx^\nu=0\,. \tag{81}$$

Equations of parallel transfer (81) are in generalized form. To use them to calculate real physical properties observed in practice, the equations should contain physical observable values (chronometric invariants). To bring the equations to the desired form we project the initial ones onto time and onto space in an accompanying frame of reference

$$\left.\begin{aligned}
&b_\alpha DP^\alpha=\sqrt{g_{00}}\left(dP^0+\Gamma^0_{\mu\nu}P^\mu dx^\nu\right)+\frac{g_{0i}}{\sqrt{g_{00}}}\left(dP^i+\Gamma^i_{\mu\nu}P^\mu dx^\nu\right)=0\,,\\
&h^i_\beta DP^\beta=dP^i+\Gamma^i_{\mu\nu}P^\mu dx^\nu=0\,.
\end{aligned}\right\} \tag{82}$$

But Christoffel symbols of the 2nd kind $\Gamma^\alpha_{\mu\nu}$ found in these equations are not yet expressed through chronometrically invariant values. We express 2nd kind Christoffel symbols $\Gamma^\alpha_{\mu\nu}$ and 1st kind Christoffel symbols $\Gamma_{\mu\nu,\sigma}$, included into them,

$$\Gamma^\alpha_{\mu\nu}=g^{\alpha\sigma}\Gamma_{\mu\nu,\sigma}\,,\quad \Gamma_{\mu\nu,\rho}=\frac{1}{2}\left(\frac{\partial g_{\mu\rho}}{\partial x^\nu}+\frac{\partial g_{\nu\rho}}{\partial x^\mu}-\frac{\partial g_{\mu\nu}}{\partial x^\rho}\right) \tag{83}$$

through chronometrically invariant properties of the frame of reference. Expressing components $g^{\alpha\beta}$ and the first derivatives from $g_{\alpha\beta}$ through F_i, A_{ik}, D_{ik}, w, and v_i after some algebra we obtain

$$\Gamma_{00,0}=-\frac{1}{c^3}\left(1-\frac{w}{c^2}\right)\frac{\partial w}{\partial t}\,, \tag{84}$$

$$\Gamma_{00,i}=\frac{1}{c^2}\left(1-\frac{w}{c^2}\right)^2F_i+\frac{1}{c^4}v_i\frac{\partial w}{\partial t}\,, \tag{85}$$

$$\Gamma_{0i,0}=-\frac{1}{c^2}\left(1-\frac{w}{c^2}\right)\frac{\partial w}{\partial x^i}\,, \tag{86}$$

$$\Gamma_{0i,j}=-\frac{1}{c}\left(1-\frac{w}{c^2}\right)\left(D_{ij}+A_{ij}+\frac{1}{c^2}F_jv_i\right)+\frac{1}{c^3}v_j\frac{\partial w}{\partial x^i}\,, \tag{87}$$

$$\Gamma_{ij,0}=\frac{1}{c}\left(1{-}\frac{w}{c^2}\right)\left[D_{ij}{-}\frac{1}{2}\left(\frac{\partial v_j}{\partial x^i}{+}\frac{\partial v_i}{\partial x^j}\right)+\frac{1}{2c^2}\left(F_iv_j{+}F_jv_i\right)\right], \tag{88}$$

$$\Gamma_{ij,k} = -\triangle_{ij,k} + \frac{1}{c^2}\left\{v_i A_{jk} + v_j A_{ik} + \frac{1}{2}v_k\left(\frac{\partial v_j}{\partial x^i} + \frac{\partial v_i}{\partial x^j}\right) - \right.$$
$$\left. -\frac{1}{2c^2}v_k\left(F_i v_j + F_j v_i\right)\right\} + \frac{1}{c^4}F_k v_i v_j\,, \tag{89}$$

$$\Gamma_{00}^{0} = -\frac{1}{c^3}\left[\frac{1}{1-\frac{w}{c^2}}\frac{\partial w}{\partial t} + \left(1-\frac{w}{c^2}\right)v_k F^k\right], \tag{90}$$

$$\Gamma_{00}^{k} = -\frac{1}{c^2}\left(1-\frac{w}{c^2}\right)^2 F^k, \tag{91}$$

$$\Gamma_{0i}^{0} = \frac{1}{c^2}\left[-\frac{1}{1-\frac{w}{c^2}}\frac{\partial w}{\partial x^i} + v_k\left(D_i^k + A_{i\cdot}^{\cdot k} + \frac{1}{c^2}v_i F^k\right)\right], \tag{92}$$

$$\Gamma_{0i}^{k} = \frac{1}{c}\left(1-\frac{w}{c^2}\right)\left(D_i^k + A_{i\cdot}^{\cdot k} + \frac{1}{c^2}v_i F^k\right), \tag{93}$$

$$\Gamma_{ij}^{0} = -\frac{1}{c}\frac{1}{1-\frac{w}{c^2}}\left\{-D_{ij} + \frac{1}{c^2}v_n\left[v_j\left(D_i^n + A_{i\cdot}^{\cdot n}\right) + v_i\left(D_j^n + A_{j\cdot}^{\cdot n}\right) + \frac{1}{c^2}v_i v_j F^n\right] + \right.$$
$$\left. + \frac{1}{2}\left(\frac{\partial v_i}{\partial x^j} + \frac{\partial v_j}{\partial x^i}\right) - \frac{1}{2c^2}\left(F_i v_j + F_j v_i\right) - \triangle_{ij}^n v_n\right\}, \tag{94}$$

$$\Gamma_{ij}^{k} = \triangle_{ij}^k - \frac{1}{c^2}\left[v_i\left(D_j^k + A_{j\cdot}^{\cdot k}\right) + v_j\left(D_i^k + A_{i\cdot}^{\cdot k}\right) + \frac{1}{c^2}v_i v_j F^k\right]. \tag{95}$$

Here $\triangle_{jk}^i$ stands for chronometrically invariant Christoffel symbols, which are defined similarly to $\Gamma_{\mu\nu}^{\alpha}$. The only difference is that here instead of $g_{\alpha\beta}$ chronometrically invariant metric tensor h_{ik} is used, i. e.

$$\triangle_{jk}^i = h^{im}\triangle_{jk,m} = \frac{1}{2}h^{im}\left(\frac{{}^*\partial h_{jm}}{\partial x^k} + \frac{{}^*\partial h_{km}}{\partial x^j} - \frac{{}^*\partial h_{jk}}{\partial x^m}\right). \tag{96}$$

Having regular operators of derivation expressed through chronometrically invariant ones and putting down $dx^0{=}cdt$ through $d\tau$ (13), we obtain a chronometrically invariant formula for a regular differential

$$d = \frac{\partial}{\partial x^\alpha}dx^\alpha = \frac{{}^*\partial}{\partial t}d\tau + \frac{{}^*\partial}{\partial x^i}dx^i. \tag{97}$$

Now having physical observable components of P^α denoted as

$$\frac{P_0}{\sqrt{g_{00}}} = \varphi\,, \qquad P^i = q^i, \tag{98}$$

we obtain its other components

$$P^0 = \frac{1}{\sqrt{g_{00}}}\left(\varphi + \frac{1}{c}v_k q^k\right), \qquad P_i = -\frac{\varphi}{c}v_i - q_i\,. \tag{99}$$

Having the obtained formulas substituted into (82) we arrive to chronometrically invariant equations of parallel transfer of vector P^α

$$\begin{aligned} &d\varphi + \frac{1}{c}\left(F_i q^i d\tau + D_{ik}q^i dx^k\right) = 0\,, \\ &dq^i + \left(\frac{\varphi}{c}dx^k + q^k d\tau\right)\left(D_k^i + A_{k\cdot}^{\cdot i}\right) - \frac{\varphi}{c}F^i d\tau + \triangle_{mk}^i q^m dx^k = 0\,. \end{aligned} \tag{100}$$

From the obtained generalized equations (100) we can make an easy transition to particular dynamic equations of motion, having φ and q^i for different types of particles substituted into the generalized equations (100) and divided by dt.

6.2 Generalized space-time

The corpuscular and wave forms of the universal dynamic vector P^α for this case were obtained in Section 5.

6.2.1 Equations of motion of real mass-bearing particles

From (63) and (64) we obtain for the corpuscular form of the universal dynamic vector P^α in case of real mass-bearing particles

$$\varphi = M\left[1-\frac{1}{c^2}\left(w+v_k u^k\right)\right], \qquad q^i = M\frac{u^i}{c}\,, \tag{101}$$

where $\dfrac{u^2}{\left[1-\frac{1}{c^2}\left(w+v_k u^k\right)\right]^2}<c^2$, $d\tau\neq 0$, $dt\neq 0$.

From here we immediately arrive to the corpuscular form of dynamic equations of motion for real mass-bearing particles

$$\begin{gathered}\frac{d}{dt}\left\{M\left[1-\frac{1}{c^2}\left(w+v_k u^k\right)\right]\right\}-\frac{M}{c^2}\left[1-\frac{1}{c^2}\left(w+v_k u^k\right)\right]F_i u^i+\frac{M}{c^2}D_{ik}u^i u^k=0\,,\\ \frac{d}{dt}\left(Mu^i\right)+2M\left[1-\frac{1}{c^2}\left(w+v_k u^k\right)\right]\left(D^i_n+A^{\cdot i}_{n\cdot}\right)u^n-\\ -M\left[1-\frac{1}{c^2}\left(w+v_k u^k\right)\right]F^i+M\triangle^i_{nk}u^n u^k=0\,,\end{gathered}\tag{102}$$

where $d=\frac{{}^*\partial}{\partial t}d\tau+\frac{{}^*\partial}{\partial x^i}dx^i$, $\frac{d}{d\tau}=\frac{{}^*\partial}{\partial t}+\mathrm{v}^i\frac{{}^*\partial}{\partial x^i}$, and

$$\frac{d}{dt}=\frac{{}^*\partial}{\partial t}\frac{d\tau}{dt}+u^i\frac{{}^*\partial}{\partial x^i}=\left[1-\frac{1}{c^2}\left(w+v_m u^m\right)\right]\frac{{}^*\partial}{\partial t}+u^i\frac{{}^*\partial}{\partial x^i}\,. \tag{103}$$

For the wave form of the universal dynamic vector P^α in case of real mass-bearing particles and according to (68) and (70) we obtain

$$\varphi=\frac{\hbar}{c^2}\frac{{}^*\partial\psi}{\partial t}\,, \qquad q^i=-\frac{\hbar}{c}h^{ik}\frac{{}^*\partial\psi}{\partial x^k}\,, \tag{104}$$

where change of wave phase with time $\frac{{}^*\partial\psi}{\partial t}$ is positive for the particles that move from past into future and is negative for those that move from future into past. From here we arrive to the wave form of (102), i. e. to dynamic equations of propagation of waves, which correspond to real mass-bearing particles within the wave-particle concept

$$\begin{gathered}\pm\frac{d}{d\tau}\left(\frac{{}^*\partial\psi}{\partial t}\right)+\left[1-\frac{1}{c^2}\left(w+v_p u^p\right)\right]F^i\frac{{}^*\partial\psi}{\partial x^i}-D^i_k u^k\frac{{}^*\partial\psi}{\partial x^i}=0\,,\\ \frac{d}{d\tau}\left(h^{ik}\frac{{}^*\partial\psi}{\partial x^k}\right)\pm\frac{1}{c^2}\left[1-\frac{1}{c^2}\left(w+v_p u^p\right)\right]\frac{{}^*\partial\psi}{\partial t}F^i-\\ -\left\{\pm\frac{1}{c^2}\frac{{}^*\partial\psi}{\partial t}u^k-h^{km}\left[1-\frac{1}{c^2}\left(w+v_p u^p\right)\right]\frac{{}^*\partial\psi}{\partial x^m}\right\}\left(D^i_k+A^{\cdot i}_{k\cdot}\right)+h^{mn}\triangle^i_{mk}u^k\frac{{}^*\partial\psi}{\partial x^n}=0\,.\end{gathered}\tag{105}$$

As you can see, the first term in the temporal component of the obtained equations (105) and two terms in the spatial components are positive for particles (waves) that move from past into future. Those components are negative for motion from future into past.

6.2.2 Equations of motion of imaginary mass-bearing particles

For this case φ and q^i in the corpuscular form of P^α will be only different from those presented for real mass-bearing particles (101) by presence of a multiplier $i=\sqrt{-1}$

$$\varphi = iM\left[1-\frac{1}{c^2}\left(w+v_k u^k\right)\right], \qquad q^i = iM\frac{u^i}{c}\,, \tag{106}$$

where $\dfrac{u^2}{\left[1-\dfrac{1}{c^2}\left(w+v_k u^k\right)\right]^2}>c^2$, $d\tau\neq 0$, $dt\neq 0$.

Respectively, the corpuscular form of dynamic equations of motion for imaginary mass-bearing particles (super-light speed particles — tachyons) will differ from similar equations for real (sub-light speed) particles (102) by presence of coefficient i at the mass term M.

The values φ and q^i for wave the form of dynamic vector of imaginary mass-bearing particles are the same as those for real particles (104). Hence the wave form of dynamic equations of motion of imaginary particles (within the wave-particle dual concept) will be similar to the wave form of equations for real particles (105).

6.2.3 Equations of motion of massless particles

According to (57) for massless (light-like) particles in generalized space-time at $v^2=c^2$

$$\frac{u^2}{\left[1-\frac{1}{c^2}\left(w+v_k u^k\right)\right]^2}=c^2\,, \qquad d\tau\neq 0\,, \quad dt\neq 0\,. \tag{107}$$

Having this formula substituted into φ and q^i for real mass-bearing particles (101) we obtain the same values for the corpuscular form of the universal dynamic vector P^α of massless particles

$$\varphi = M\frac{u}{c}\,, \qquad q^i = M\frac{u^i}{c}\,. \tag{108}$$

Respectively, the corpuscular form of dynamic equations of motion for massless particles is

$$\begin{aligned}
&\frac{d}{dt}\left(Mu\right)+\frac{Mu}{c^2}F_i u^i+\frac{M}{c}D_{ik}u^i u^k=0\,,\\
&\frac{d}{dt}\left(Mu^i\right)+2M\frac{u}{c}\left(D^i_n+A^{\cdot i}_{n\cdot}\right)u^n-M\frac{u}{c}F^i+M\triangle^i_{nk}u^n u^k=0\,.
\end{aligned} \tag{109}$$

The values φ and q^i for the wave form of massless particles will be similar to the wave form of φ and q^i of mass-bearing particles (104). Respectively, dynamic equations of propagation of waves, which correspond to massless particles within de Broglie's wave-particle concept, will be similar to formulas (105).

6.2.4 Equations of motion of zero-particles

In degenerated space-time, i.e. under the conditions of degeneration, components φ and q^i in the corpuscular for of generalized dynamic vector P^α become

$$\varphi = 0\,, \qquad q^i = M\frac{u^i}{c}\,, \tag{110}$$

where $w+v_k u^k=c^2$, $d\tau=0$, $dt\neq 0$. From here we can obtain the corpuscular form of dynamic equations of motion for zero-particles

$$\frac{M}{c^2}D_{ik}u^i u^k=0\,, \qquad \frac{d}{dt}\left(Mu^i\right)+M\triangle^i_{nk}u^n u^k=0\,. \tag{111}$$

The physical observable components φ and q^i for the wave form of generalized dynamic vector P^α in degenerated space-time are

$$\varphi = 0\,, \qquad q^i = -\frac{\hbar}{c}h^{ik}\frac{^*\partial\psi}{\partial x^k}\,, \tag{112}$$

from which we arrive to the wave form of dynamic equations of motion of zero-particles

$$D^m_k u^k\frac{^*\partial\psi}{\partial x^m}=0\,, \qquad \frac{d}{dt}\left(h^{ik}\frac{^*\partial\psi}{\partial x^k}\right)+h^{mn}\triangle^i_{mk}u^k\frac{^*\partial\psi}{\partial x^n}=0\,, \tag{113}$$

i.e. dynamic equations of propagation of waves that correspond to zero-particles within de Broglie's wave-particle concept.

6.3 Strictly non-degenerated space-time

For this case the corpuscular and the wave forms of the universal dynamic vector P^{α} were obtained earlier in Section 3.

6.3.1 Equations of motion of real mass-bearing particles

According to (37) and (26) for the corpuscular form of the universal dynamic vector P^{α} for real mass-bearing particles we have

$$\varphi = \pm m\,, \qquad q^i = \frac{1}{c}m\mathrm{v}^i, \tag{114}$$

where $\mathrm{v}^2{<}c^2$, $d\tau{\neq}0$, $dt{\neq}0$. Using these values we obtain dynamic equations of motion of particles with positive relativistic mass $m{>}0$ (which move from past into future)

$$\begin{aligned} &\frac{dm}{d\tau} - \frac{m}{c^2}F_i\mathrm{v}^i + \frac{m}{c^2}D_{ik}\mathrm{v}^i\mathrm{v}^k = 0\,, \\ &\frac{d\left(m\mathrm{v}^i\right)}{d\tau} + 2m\left(D^i_k + A^{\cdot i}_{k\cdot}\right)\mathrm{v}^k - mF^i + m\triangle^i_{nk}\mathrm{v}^n\mathrm{v}^k = 0\,, \end{aligned} \tag{115}$$

as well as for particles with negative mass $m{<}0$ (which move into past)

$$\begin{aligned} &-\frac{dm}{d\tau} - \frac{m}{c^2}F_i\mathrm{v}^i + \frac{m}{c^2}D_{ik}\mathrm{v}^i\mathrm{v}^k = 0\,, \\ &\frac{d\left(m\mathrm{v}^i\right)}{d\tau} + mF^i + m\triangle^i_{nk}\mathrm{v}^n\mathrm{v}^k = 0\,. \end{aligned} \tag{116}$$

For the wave form of the universal dynamic vector P^{α} from (42) and (45) we obtain formulas that are similar to those obtained earlier for φ and q^i in generalized space-time (104)

$$\varphi = \frac{\hbar}{c^2}\frac{{}^*\partial\psi}{\partial t}\,, \qquad q^i = -\frac{\hbar}{c}h^{ik}\frac{{}^*\partial\psi}{\partial x^k}\,, \tag{117}$$

where $\frac{{}^*\partial\psi}{\partial t}$ i. e. change of phase with time, is positive for motion of particles from past into future and is negative for motion of particles into past. Taking into account that the chronometrically invariant equations of parallel transfer of P^{α} (100) in strictly non-degenerated space-time are divided by interval of observable time $d\tau$, we obtain the wave form of dynamic equations of motion of mass-bearing real particles

$$\begin{aligned} &\pm\frac{d}{d\tau}\left(\frac{{}^*\partial\psi}{\partial t}\right) + F^i\frac{{}^*\partial\psi}{\partial x^i} - D^i_k\mathrm{v}^k\frac{{}^*\partial\psi}{\partial x^i} = 0\,, \\ &\frac{d}{d\tau}\left(h^{ik}\frac{{}^*\partial\psi}{\partial x^k}\right) - \left(D^i_k + A^{\cdot i}_{k\cdot}\right)\left(\pm\frac{1}{c^2}\frac{{}^*\partial\psi}{\partial t}\mathrm{v}^k - h^{km}\frac{{}^*\partial\psi}{\partial x^m}\right) \pm \\ &\qquad\qquad \pm\frac{1}{c^2}\frac{{}^*\partial\psi}{\partial t}F^i + h^{mn}\triangle^i_{mk}\mathrm{v}^k\frac{{}^*\partial\psi}{\partial x^n} = 0\,. \end{aligned} \tag{118}$$

The first term of the temporal projection and the first two terms of the spatial projections are positive for motion of particles from past into future. These terms are negative for motion from future into past.

6.3.2 Equations of motion of imaginary mass-bearing particles

In this case the corpuscular forms of φ and q^i will be different from those of φ and q^i for real mass-bearing particles (114) by presence of $i{=}\sqrt{-1}$

$$\varphi = \pm im\,, \qquad q^i = i\frac{1}{c}m\mathrm{v}^i, \tag{119}$$

where $\mathrm{v}^2{>}c^2$, $d\tau{\neq}0$, $dt{\neq}0$. Respectively, the corpuscular form of dynamic equations of motion of imaginary (super-light speed) particles will be different from those we have obtained for real (sub-light speed) particles by presence of coefficient i and mass term m.

The wave forms of φ and q^i for imaginary mass-bearing particles will be similar to the wave forms of φ and q^i for real mass-bearing particles (117). Respectively, dynamic equations of propagation of waves, which correspond to imaginary mass-bearing particles, will be similar to dynamic equations of propagation of waves, which correspond to real mass-bearing particles (118).

We see that from the viewpoint of the wave concept there is no difference at what speed a mass-bearing particle travels (or the wave propagates) — below the speed of light or above that. To the contrary, from the viewpoint of the corpuscular concept there is difference, because the corpuscular equations of motion of super-light speed (imaginary) particles differ from those of sub-light speed ones by presence of coefficient i at the mass term m.

6.3.3 Equations of motion of massless particles

In this case the corpuscular forms of φ and q^i becomes

$$\varphi = \pm\frac{\omega}{c} = \pm k\,, \qquad q^i = \frac{1}{c}k\mathrm{v}^i = \frac{1}{c}kc^i, \tag{120}$$

where $\mathrm{v}^2{=}c^2$, $d\tau{\neq}0$, $dt{\neq}0$ and chronometrically invariant (physical observable) vector of three-dimensional velocity of particle v^i equals to chronometrically invariant (observable) three-dimensional vector of light velocity c^i

$$\mathrm{v}^i = \frac{dx^i}{d\tau} = c^i. \tag{121}$$

Respectively, the corpuscular dynamic equations of motion of massless particles are: for massless particles that bear positive relativistic frequency $\omega{>}0$ and travel from past into future

$$\left.\begin{aligned} &\frac{d\omega}{d\tau} - \frac{\omega}{c^2}F_i c^i + \frac{\omega}{c^2}D_{ik}c^i c^k = 0\,, \\ &\frac{d\left(\omega c^i\right)}{d\tau} + 2\omega\left(D^i_k + A^{\cdot i}_{k\cdot}\right)c^k - \omega F^i + \omega\triangle^i_{nk}c^n c^k = 0\,. \end{aligned}\right\} \tag{122}$$

For massless particles that bear $\omega{<}0$ and travel from future into past

$$\left.\begin{aligned} &-\frac{d\omega}{d\tau} - \frac{\omega}{c^2}F_i c^i + \frac{\omega}{c^2}D_{ik}c^i c^k = 0\,, \\ &\frac{d\left(\omega c^i\right)}{d\tau} + \omega F^i + \omega\triangle^i_{nk}c^n c^k = 0\,. \end{aligned}\right\} \tag{123}$$

The wave forms of φ and q^i for massless particles are similar to those of φ and q^i for mass-bearing particles (117). Respectively, dynamic equations of propagation of waves, which correspond to massless (light-like) particles within the wave-particle dual concept, will be similar too. The only difference will be in vector of observable velocity of light c^i, which will replace vector of observable velocity of particle v^i.

6.4 Specific case: geodesic equations

What are geodesic equations? As we mentioned in Section 1, those are kinematic equations of motion of particles along the shortest (geodesic) trajectories. From geometric viewpoint geodesic equations are those of Levi-Civita parallel transfer

$$\frac{DQ^\alpha}{d\rho} = \frac{dQ^\alpha}{d\rho} + \Gamma^\alpha_{\mu\nu}Q^\mu\frac{dx^\nu}{d\rho} = \frac{d^2x^\alpha}{d\rho^2} + \Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{d\rho}\frac{dx^\nu}{d\rho} = 0 \tag{124}$$

of four-dimensional kinematic vector of particle $Q^\alpha{=}\frac{dx^\alpha}{d\rho}$, which is tangential to the trajectory at its every point. Respectively, non-isotropic geodesic equations (trajectories of mass-bearing free particles) and isotropic geodesic equations (massless free particles) are

$$\frac{DQ^\alpha}{ds} = \frac{d^2x^\alpha}{ds^2} + \Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0\,, \qquad Q^\alpha = \frac{dx^\alpha}{ds}\,, \tag{125}$$

$$\frac{DQ^\alpha}{d\sigma} = \frac{d^2x^\alpha}{d\sigma^2} + \Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\sigma} = 0\,, \qquad Q^\alpha = \frac{dx^\alpha}{d\sigma}\,. \tag{126}$$

But any kinematic vector, similar to dynamic vector P^α of mass-bearing particles and to wave vector K^α of massless particles, is a partial case of an arbitrary vector Q^α, for which we have obtained universal equations of motion. Hence having values φ and q^i for kinematic vector of mass-bearing particles substituted into the universal equations of motion (100), we will immediately arrive to non-isotropic geodesic equations in chronometrically invariant form. Similarly, having substituted φ and q^i for kinematic vector of massless particles we will arrive to chronometrically invariant isotropic geodesic equations. This is what we are going to do now.

For kinematic vector of mass-bearing particles we have

$$\begin{aligned} \varphi &= \frac{Q_0}{\sqrt{g_{00}}} = \frac{g_{0\alpha}Q^\alpha}{\sqrt{g_{00}}} = \pm\frac{1}{\sqrt{1-\mathrm{v}^2/c^2}}\,, \\ q^i &= Q^i = \frac{dx^i}{ds} = \frac{1}{\sqrt{1-\mathrm{v}^2/c^2}}\frac{dx^i}{cd\tau} = \frac{1}{c\sqrt{1-\mathrm{v}^2/c^2}}\mathrm{v}^i. \end{aligned} \tag{127}$$

For massless particles, taking into account that for isotropic trajectories $d\sigma{=}cd\tau$ we have

$$\begin{aligned} \varphi &= \sqrt{g_{00}}\frac{dx^0}{d\sigma} + \frac{1}{c\sqrt{g_{00}}}g_{0i}c^i = \pm 1\,, \\ q^i &= \frac{dx^i}{d\sigma} = \frac{dx^i}{cd\tau} = \frac{1}{c}\,c^i. \end{aligned} \tag{128}$$

Having these values substituted into generalized equations of motion (100), we obtain *chronometrically invariant geodesic equations for mass-bearing particles* (non-isotropic geodesic equations)

$$\begin{aligned} &\pm\frac{d}{d\tau}\left(\frac{1}{\sqrt{1-\mathrm{v}^2/c^2}}\right) - \frac{F_i\mathrm{v}^i}{c^2\sqrt{1-\mathrm{v}^2/c^2}} + \frac{D_{ik}\mathrm{v}^i\mathrm{v}^k}{c^2\sqrt{1-\mathrm{v}^2/c^2}} = 0\,, \\ &\frac{d}{d\tau}\left(\frac{\mathrm{v}^i}{\sqrt{1-\mathrm{v}^2/c^2}}\right) \mp \frac{F^i}{\sqrt{1-\mathrm{v}^2/c^2}} + \frac{\triangle^i_{nk}\mathrm{v}^n\mathrm{v}^k}{\sqrt{1-\mathrm{v}^2/c^2}} + \frac{(1\pm 1)}{\sqrt{1-\mathrm{v}^2/c^2}}\left(D^i_k + A^{\cdot i}_{k\cdot}\right)\mathrm{v}^k = 0\,, \end{aligned} \tag{129}$$

as well as equations for *massless particles* (isotropic geodesic equations)

$$\begin{aligned} &D_{ik}c^ic^k - F_ic^i = 0\,, \\ &\frac{dc^i}{d\tau} \mp F^i + \triangle^i_{nk}c^nc^k + (1\pm 1)\left(D^i_k + A^{\cdot i}_{k\cdot}\right)c^k = 0\,. \end{aligned} \tag{130}$$

The upper sign in the alternating terms in these equations stands for motion of particles into past, while the lower sign stands for motion into past. As seen, we again have the asymmetry of motion in time. The same asymmetry was observed in dynamic equations of motion. We see that this asymmetry does not depend upon physical properties of particles themselves, but rather upon properties of the observer's space (body) of reference, i.e. upon F^i, A_{ik}, and D_{ik}. In absence of gravitational inertial forces, rotation or deformation of the space of reference, the asymmetry is absent too.

6.5 Specific case: Newton laws

In this Section we are going to prove that chronometrically invariant dynamic equations of motion of mass-bearing particles are four-dimensional generalization of Newton's 1st and 2nd laws in space-time, where gravitational inertial force F^i, rotation A_{ik}, or deformation D_{ik} are present.

At low speed $m{=}m_0$ and dynamic equations of motion become

$$\frac{DP^\alpha}{ds} = m_0\frac{d^2x^\alpha}{ds^2} + m_0\Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0\,, \qquad P^\alpha = \frac{dx^\alpha}{ds}\,, \tag{131}$$

having the equations divided by m_0, dynamic equations turn immediately into kinematic ones, i.e. regular non-isotropic geodesic equations.

These are dynamic equations of motion of so-called "free particles", i. e. particles that fall freely under action of gravitational field.

Motion of particles under action of an additional force R^α not of gravitational nature, is not geodesic and their equations of motion become

$$m_0\frac{d^2x^\alpha}{ds^2}+m_0\Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}=R^\alpha. \tag{132}$$

But these are dynamic equations of motion of particles in *four-dimensional space-time*, while Newton laws were set forth for *three-dimensional space*. In particular, the derivation parameter in these equations is space-time interval, not applicable to three-dimensional space.

Let us now look at chronometrically invariant dynamic equations of motion of mass-bearing particles. At low speed these are

$$\frac{m_0}{c^2}\left(D_{ik}\mathrm{v}^i\mathrm{v}^k-F_i\mathrm{v}^i\right)=0\,, \tag{133}$$

$$m_0\frac{d^2x^i}{d\tau^2}-m_0F^i+m_0\triangle^i_{nk}\mathrm{v}^n\mathrm{v}^k+2m_0\left(D^i_k+A^{\cdot i}_{k\cdot}\right)\mathrm{v}^k=0\,, \tag{134}$$

where spatial observable projections (134) are dynamic equations of motion in three-dimensional space.

In space-time with three-dimensional Euclidean metric all values $h^k_i{=}\delta^k_i$ and tensor of space deformation velocities $D_{ik}{=}\frac{1}{2}\frac{^*\partial h_{ik}}{\partial t}{=}0$. In this case $\triangle^i_{kn}{=}0$ and hence the term $m_0\triangle^i_{nk}\mathrm{v}^n\mathrm{v}^k{=}0$. If also $F^i{=}0$ and $A_{ik}{=}0$, spatial observable projections of equations of motion become

$$m_0\frac{d^2x^i}{d\tau^2}=0\,, \tag{135}$$

or in another form

$$\mathrm{v}^i=\frac{dx^i}{d\tau}=const. \tag{136}$$

Hence four-dimensional generalization of *Newton's 1st law* for mass-bearing particles can be set forth as follows:

> **If a particle is free from action of gravitational inertial forces (or such forces are balanced) and at the same time rotation or deformation of space is absent, such particle will experience straight and even motion.**

Such condition, as seen from formulas for Christoffel symbols (90–95), is only possible when all $\Gamma^\alpha_{\mu\nu}{=}0$, because any of Christoffel symbols are functions of at least one of the values F^i, A_{ik}, or D_{ik}.

Now let us assume that $F^i{\neq}0$, but $A_{ik}{=}0$ and $D_{ik}{=}0$. In this case three-dimensional dynamic equations of motion of particles become

$$\frac{d^2x^i}{d\tau^2}=F^i. \tag{137}$$

But gravitational potential and force F^i (and values A_{ik}, D_{ik}) by definition describe the body of reference itself, which is the source of gravitational field. Hence F^i describes gravitational field itself, i. e. is a force that acts on a unit-mass particle. Gravitational inertial force that acts on a particle with mass m_0 is

$$\Phi^i=m_0F^i, \tag{138}$$

and three-dimensional dynamic equations of motion become

$$m_0\frac{d^2x^i}{d\tau^2}=\Phi^i. \tag{139}$$

Respectively, four-dimensional generalization of *Newton's 2nd law* for mass-bearing particles can be set forth as follows:

> **Acceleration that a particle gains from gravitational field is proportional to gravitational inertial force that acts on the particle and is reciprocal to its mass in absence of deformation or rotation of space.**

Having any particular value of gravitational inertial force Φ^i substituted into three-dimensional equations of motion (134)

$$m_0 \frac{d^2x^i}{d\tau^2} + m_0 \triangle^i_{nk} \mathrm{v}^n \mathrm{v}^k + 2m_0 \left(D^i_k + A^{\cdot i}_{k\cdot}\right) \mathrm{v}^k = \Phi^i, \tag{140}$$

we can solve them to obtain trajectory of motion of particle in three-dimensional space, i. e. dependance of its three-dimensional coordinates from time. But presence of gravitational inertial force is not mandatory to make motion curved and uneven. As seen from the equations this happens if at least one of the values F^i, A_{ik}, or D_{ik} is not zero. Hence theoretically a particle can be in state of uneven and curved motion in absence of gravitational inertial forces, but in presence of rotation or deformation of space or both.

If particle moves under joint action of gravitational inertial force Φ^i and another non-gravitational force R^i, its three-dimensional dynamic equations of motion become

$$m_0 \frac{d^2x^i}{d\tau^2} + m_0 \triangle^i_{nk} \mathrm{v}^n \mathrm{v}^k + 2m_0 \left(D^i_k + A^{\cdot i}_{k\cdot}\right) \mathrm{v}^k = \Phi^i + R^i. \tag{141}$$

In a flat three-dimensional space, as we have mentioned in the above, $\triangle^i_{kn}$=0 and the second term in these equations will be zero.

If in a *flat* three-dimensional space rotation or deformation are absent, dynamic equations of motion of particles become very simple

$$m_0 \frac{d^2x^i}{d\tau^2} = \Phi^i, \qquad\qquad m_0 \frac{d^2x^i}{d\tau^2} = \Phi^i + R^i, \tag{142}$$

to describe motion under action of gravitational inertial force Φ^i (the first equations) as well as motion under joint action of Φ^i and another non-gravitational force R^i which deviates particles from geodesic line (the second equations).

We obtained, that motion under action of gravitational inertial forces is possible in either curved or flat space. Why?

As known, curvature of space-time is characterized by Riemann-Christoffel tensor of curvature $R_{\alpha\beta\gamma\delta}$, which depends upon the second derivatives of $g_{\alpha\beta}$ and the first derivatives of $g_{\alpha\beta}$. The condition $R_{\alpha\beta\gamma\delta}\neq 0$ is the necessary and sufficient condition of curved space. To have non-zero curvature of space-time it is necessary and sufficient that the second derivatives of metric $g_{\alpha\beta}$ are non-zeroes.

But we also know, that the first derivatives of space-time metric $g_{\alpha\beta}$ in a flat space-time may be not equal to zero.

Our chronometrically invariant equations of motion contain values $\triangle^i_{kn}$, F^i, A_{ik}, and D_{ik}, which depend upon the first derivatives of metric tensor only. Therefore at $R_{\alpha\beta\gamma\delta}$=0 (flat space-time) gravitational inertial force F^i, rotation A_{ik}, and deformation of space D_{ik}, which are functions of the first derivatives of metric $g_{\alpha\beta}$ may be not equal to zero.

6.6 Resume

So, did we get?

First, using method of chronometric invariants we have obtained, that in General Relativity space-time particles may move not only into future (in respect to us), but into past as well. Besides, physical observable values for mass-bearing particles are relativistic mass $\pm m$ and three-dimensional value $\frac{1}{c}m\mathrm{v}^i$, where plus stands for motion of particle into future, while minus stands for motion into past. For massless particles physical observable values are wave number $\pm k = \pm\frac{\omega}{c}$ and three-dimensional value $\frac{1}{c}kc^i$. Besides, motions of particles into future and into past are asymmetric in respect to each other. This asymmetry depends upon properties of space only (i. e. gravitational inertial force F^i, rotation A_{ik}, and deformation D_{ik}).

Further, looking at motion of particles as propagation of waves (within de Broglie's wave-particle dual concept), we observe no asymmetry. From observer's viewpoint propagation of waves is the same in both directions in time, while movement of particle "balls" is not.

Second, we have shown that chronometrically invariant dynamic equations of motion are generalization of Newton's laws in four-dimensional space-time, where gravitational inertial force F^i, rotation

A_{ik}, or deformation D_{ik} are present. Motion of particles is straight and even only if F^i=0, A_{ik}=0, and D_{ik}=0. If any of the above values is not zero, motion of a particle is no longer straight and even. Besides, curved and uneven motion may take place in a flat space too, of course provided at least one of the values F^i, A_{ik}, or D_{ik} is not zero.

All the above results have been obtained exclusively thanks to Zelmanov's method of chronometric invariants (the mathematical apparatus of physical observable values). Regular general covariant methods would be of no use here.

As a result we can see that not all physical effects in General Relativity are yet known in contemporary science. Further development of experimental physics and astronomy will discover new phenomena, related, in particular, to acceleration, rotation and deformation of space of reference.

7 Analysis of the equations

7.1 Space-time and zero-space: limit transitions

As we can see, at $w=-v_i u^i$ in our formulas values in generalized space-time ($g\leq 0$) are replaced by those in non-degenerated space-time ($g<0$)

$$d\tau = \left[1 - \frac{1}{c^2}\left(w + v_i u^i\right)\right] dt = dt\,, \tag{143}$$

$$u^i = \frac{dx^i}{dt} = \frac{dx^i}{d\tau} = \mathrm{v}^i, \tag{144}$$

$$M = \frac{m}{1 - \frac{1}{c^2}\left(w + v_i u^i\right)} = m\,, \tag{145}$$

$$P^0 = M = m\,, \qquad P^i = \frac{1}{c} M u^i = \frac{1}{c} m \mathrm{v}^i, \tag{146}$$

and in this transition coordinate time t coincides to observable time τ.

Of course this transformation also occurs under a narrower condition $w=-v_i u^i$=0, when $w\to 0$ (weak gravitational field) and v_i=0 (no rotation of space) at the same time. But in the observed part of the Universe there is hardly an area where rotation and gravitational filed are absolutely absent. Therefore we see that transition to regular (strictly non-degenerated) space-time always happens at

$$w = -v_i u^i = -v_i \mathrm{v}^i. \tag{147}$$

Corpuscular equations of motion of mass-bearing and massless particles in generalized space-time transform into the equations in regular (strictly non-degenerated) space-time only in case of motion from past into future (direct flow of time). That is, only for real mass-bearing particles with $m>0$, imaginary mass-bearing particles with $im>0$ and for massless particles with $\omega>0$.

Equations of motion in wave form in generalized space-time transform into the equations in regular space-time in the same way for particles with $m>0$, $im>0$ and $\omega>0$ (direct flow of time), and for particles with $m<0$, $im<0$ and $\omega<0$ (reverse flow of time).

Later we are going to find out *why* does that happen.

In regular space-time ($g<0$) we obtained P^0 (26), which after substitution of $\frac{dt}{d\tau}$ (32) and the transition conditions $w=-v_i u^i=-v_i \mathrm{v}^i$ takes sign-alternating relativistic mass, i. e. values $+m$ and $-m$

$$P^0 = m\frac{dt}{d\tau} = \frac{m}{1-\frac{w}{c^2}}\left(\frac{1}{c^2}v_i \mathrm{v}^i \pm 1\right) = \pm m\,. \tag{148}$$

In generalized space-time $g\leq 0$ we obtained $P^0=M$, but through another method (62) without using $\frac{dt}{d\tau}$, due to which the formula accepts alternating sign.

But in such case the component $P^0=\pm m$ in regular space-time (148), taking two values, can not be a partial case of a single value $P^0=M$ in generalized space-time.

To understand the reason we turn from sign-alternating $P^0 = \pm m$ in regular space-time to $P^0 = M$ in generalized space-time. This can be easily done by substituting the already known relationship between physical observable velocity v^i and its coordinate velocity u^i (57), into sign-alternating formula $P^0 = \pm m$ (148).

As a result we obtain expanded relationship for the component P^0 in generalized space-time

$$P^0 = \frac{m}{1 - \frac{w}{c^2}} \left[\frac{1}{c^2} \frac{v_i u^i}{1 - \frac{1}{c^2}\left(w + v_i u^i\right)} \pm 1 \right], \tag{149}$$

which evidently accepts alternating sign. For particles that move in generalized space-time from past into future P^0 becomes

$$P^0 = \frac{m}{1 - \frac{1}{c^2}\left(w + v_i u^i\right)} = +M\,, \tag{150}$$

which is similar to (62). For particles that travel from future into past, P^0 becomes

$$P^0 = \frac{m \left[\frac{1}{c^2}\left(2 v_i u^i + w\right) - 1 \right]}{\left(1 - \frac{w}{c^2}\right) \left[1 - \frac{1}{c^2}\left(w + v_i u^i\right) \right]} = -M\,. \tag{151}$$

The obtained P^0 values are final generalized equations, because in transition to regular space-time the first value $P^0 = +M$ (150) unambiguously transforms into $P^0 = +m$, while the second value $P^0 = -M$ (151) transforms into $P^0 = -m$.

Noteworthy, our remarks in respect to sign-alternating value P^0 do not affect correctness of the obtained equations of motion, because those include gravitational rotational mass in general notation M without any respect to its particular composition. The only difference is that sequential substitution of the two values of M into equations of motion in generalized space-time will produce independent equations: for particles that move from past into future and for those that move from future into past.

Let us now return to physical condition $w = -v_i u^i$ (147), which marks the transition from dynamic equations of motion in generalized space-time to those in regular space-time. We have also seen that under this condition $d\tau = dt$ (143). But we know that in regular space-time the equality $d\tau = dt$ is not imperative. To the contrary, in the observed Universe interval of physical observable time $d\tau$ is almost always a bit different from interval of coordinate time dt.

Therefore transition from generalized space-time to regular space-time occurs under physical conditions $w = -v_i u^i$, which are merely a partial case of physical conditions in regular space-time. But from here we see that strictly non-degenerated space-time ($g<0$) is *not* an area of generalized space-time $g \leq 0$ we have looked at.

Does that contain a contradiction between equations of motion in regular space-time and in generalized space-time?

No it doesn't. All laws applicable to regular space-time ($g<0$) are as well true in non-degenerated area ($g<0$) of generalized space-time $g \leq 0$. Those two non-degenerated areas *are not* the same. That is, degenerated space-time added to regular space-time produces two absolutely separate manifolds. Generalized space-time is a different manifold absolutely independent from either strictly non-degenerated space-time or degenerated one. And there is no surprise in that transition from one to another occurs under very limited partial conditions.

The only question is what configuration of those manifolds exists in the observable Universe. Two options are possible here:

1. non-degenerated space-time ($g<0$) and degenerated space-time ($g=0$) exist as two separate manifold (regular space-time of General Relativity with a small "add-on" of zero-space);
2. non-degenerated space-time and degenerated space-time exist as two internal areas of the same manifold — generalized space-time ($g \leq 0$).

In either case transition from non-degenerated space-time into degenerated one occurs under physical condition of degeneration (56). Future experiments will show which one of these two options exists in reality.

7.2 Space-time asymmetry and world beyond the Mirror

Let us now compare corpuscular equations of motion for particles with $m>0$ (115) and $\omega>0$ (122) with those for particles with $m<0$ (116) and $\omega<0$ (123).

Even the first look at the equations shows that corpuscular equations of motion for particles with positive relativistic mass or frequency (which travel from past into future) are different from those for particles with negative mass or frequency (which travel into past). The same asymmetry exists for the wave form of such equations of motion. Why?

Asymmetry of equations of motion for particles that move into future or into past says that in four-dimensional uneven space-time there exists a fundamental asymmetry of directions from past into future and from future into past.

To understand the reasons for such fundamental asymmetry let us consider an example.

We assume that in four-dimensional space-time there exists a *Mirror* which coincides with the spatial section and hence separates past from future. We also assume that the Mirror reflects particles and waves that move either from past into future or from future into past. Then particles that travel from past into future ($m>0$, $im>0$, and $\omega>0$) always hit the Mirror and bounce back in time, i. e. into past. Consequently their properties reverse ($m<0$, $im<0$, and $\omega<0$). And *vice versa* particles and waves that travel from past future into past ($m<0$, $im<0$, and $\omega<0$) hitting the Mirror change the sign of their properties ($m>0$, $im>0$, and $\omega>0$) to bounce back into future.

Now everything becomes easy to understand. Let us look at the wave form of dynamic equations of motion (118). After reflection from the Mirror the value $\frac{^*\partial\psi}{\partial t}$ changes its sign. Hence the equations of propagation of wave into future ("plus" in the equations) become those of propagation of the same wave into past ("minus" in the equations). And *vice versa* the equations of propagation into past ("minus") after reflection become the equations of propagation into future ("plus").

Noteworthy, equations of propagation of waves from past into future and from future into past transform into each other *in full*, i. e. no terms are contracted and no terms are added. Hence the wave form of matter *fully reflects* from the Mirror.

But this is not the case for corpuscular equations. After reflection from the Mirror the values $\varphi=\pm m$ for mass-bearing particles and $\varphi=\pm k=\pm\frac{\omega}{c}$ for massless particles change their signs. But here equations of motion from past into future transform into those from future into past *not in full*.

In spatial projections of equations of motion from past into future there is an additional term present. The term is not found in spatial projections of equations of motion from past into future. For mass-bearing and massless particles the term is, respectively

$$2m\left(D^i_k + A^{\cdot i}_{k\cdot}\right)\mathrm{v}^k, \qquad 2k\left(D^i_k + A^{\cdot i}_{k\cdot}\right)c^k. \tag{152}$$

Hence we see that particle that moves from future into past hits the Mirror and bounces back to acquire an additional term in dynamic equations of motion. And *vice versa*, particle that travels from past into future bounces from the Mirror to loose a term in its dynamic equations. Therefore the Mirror itself affects trajectories of particles!

As a result, particles with negative masses or frequencies *are not* simple mirror reflections of particles with positive masses or frequencies. Either in case of motion of particles or in case of propagation of waves we do not deal with simple reflection or bouncing from the Mirror, but with *penetration* through the Mirror into the *mirror world*.

In this mirror world all particles bear negative masses or frequencies and move (from viewpoint of our world's observer) from future into past. The wave form of matter from our world has no effect on events in the mirror world, while the wave form of matter in the mirror world has no effect on events in our world. To the contrary, particles in our world may affect events in the mirror world and particles in the mirror world may have effect on events in our world.

Full isolation of our world from the mirror world, i. e. absence of mutual influence between particles of both worlds takes place under an evident condition

$$D^i_k\mathrm{v}^k = -A^{\cdot i}_{k\cdot}\mathrm{v}^k, \tag{153}$$

when the auxiliary term (152), which causes asymmetry in corpuscular equations of motion, is zero. This happens when $D^i_k=0$ and $A^{\cdot i}_{k\cdot}=0$, i. e. with full absence of deformation or rotation of the space of the body of reference.

Noteworthy, if particles with positive masses (or frequencies) co-existed in our world with those with negative masses (or frequencies), they would interfere to destroy each other inevitably and no particles would be left in our world. But we observe nothing of the kind.

Therefore in the second part of analysis of the obtained equations of motion we can make the following conclusions.

1. Fundamental asymmetry of directions from past into future and from future into past is due to existence of a certain mirror medium (the Mirror) in space-time, which fills spatial sections. All particles or waves that travel either from past or from future reflect from it. This mirror medium is degenerated space-time (zero-space);
2. Space-time falls apart into our world and the mirror world. In our world $m>0$, $im>0$, $\omega>0$, and $\frac{^*\partial\psi}{\partial t}>0$, and all particles travel from past into future. In the mirror world $m<0$, $im<0$, $\omega<0$, and $\frac{^*\partial\psi}{\partial t}<0$ and particles move from future into past;
3. Particles with $m<0$, $im<0$, $\omega<0$, and waves with $\frac{^*\partial\psi}{\partial t}<0$, which travel from future into past are particles of our world that penetrated into the mirror world through the Mirror;
4. We can not observe particles with negative masses or frequencies not waves with negative phases because they exist in the mirror world, i. e. beyond the Mirror. Particles or waves that we can observe on the exit from the Mirror (or when bouncing the Mirror, as it seems to us) have positive properties as they have come from the mirror world into our world and travel from past into future.

8 Conditions of direct and reverse flow of time

In this Section we are going to look at physical conditions under which: (a) time has direct flow, i. e. from past into future, (b) time has reverse flow, i. e. from future into past, and (c) time stops.

In contemporary physics time is defined as the fourth coordinate $x^0=ct$ of four-dimensional space-time, where c is speed of light and t is coordinate time. The structure of the formula itself says that t changes evenly with the speed of light and does not depend upon physical conditions of observation. Hence coordinate time is also referred to as *ideal time*. Aside for ideal time there is observer's *real time* τ, which depends upon conditions of observation. Theory of chronometric invariants defines interval of physical observable time as projection of increment of four-dimensional coordinates dx^α on time

$$d\tau=\frac{1}{c}b_\alpha dx^\alpha. \tag{154}$$

In the frame of reference of sub-light speed (substantional) observer, which accompanies their body of reference, interval of observable time according to (13) is

$$d\tau=\left(1-\frac{w}{c^2}\right)dt-\frac{1}{c^2}v_i dx^i=dt-\frac{1}{c^2}w dt-\frac{1}{c^2}v_i dx^i. \tag{155}$$

From here we see that $d\tau$ consists of three parts: (a) interval of coordinate time dt, (b) interval of "gravitational" time $dt_g=\frac{1}{c^2}w dt$, and (3) interval of "rotational" time $dt_{rot}=\frac{1}{c^2}v_i dx^i$. The stronger is gravitational field of the body of reference and the faster rotates the space of reference, the slower flows observer's time. Theoretically strong enough gravitational field and fast enough rotation of space may stop observer's physical time.

We define the *mirror world* as space-time where time flows backward in respect to that in reference space-time. Direction of coordinate time t, which describes displacement along temporal coordinate axis $x^0=ct$, is defined by the sign of derivative $\frac{dt}{d\tau}$. Respectively, direction of observable time τ is defined by the sign of derivative $\frac{d\tau}{dt}$.

The formula for $\frac{dt}{d\tau}$ was obtained in Section 3 from the condition of conservation of four-dimensional velocity of particle along its four-dimensional trajectory as (28–30). But it can be also obtained in another way by presenting the square of space-time interval $ds^2{=}c^2d\tau^2{-}d\sigma^2$ as

$$ds^2 = \left(1-\frac{w}{c^2}\right)^2 c^2dt^2 - 2\left(1-\frac{w}{c^2}\right)v_i dx^i dt + g_{ik}dx^i dx^k. \tag{156}$$

From here we see that the square of elementary distance between two infinitely close points in space-time is the sum of the square of three-dimensional distance $g_{ik}dx^i dx^k$ and of two terms, which depend upon physical properties of space.

The value $\left(1{-}\frac{w}{c^2}\right)^2 c^2dt^2$ is a term in ds^2 caused by presence of the fourth dimension (time) and presence of space of reference's own gravitational field w. In absence of gravitational field temporal coordinate $x^0{=}ct$ changes evenly with speed of light. But if $w{\neq}0$, coordinate x^0 changes "slower" by value $\frac{w}{c^2}$. The stronger is gravitational potential w, the slower flows coordinate time. At $w{=}c^2$ coordinate time stops at all. As well-known, such condition is implemented in gravitational collapse (black hole).

The value $\left(1{-}\frac{w}{c^2}\right)v_i dx^i dt$ is a term in ds^2 that is due to joint action of gravitational and space rotation. This term is not zero only when $w{\neq}c^2$ (no gravitational collapse) and $v_i{\neq}0$ (three-dimensional space rotates, i. e. is non-holonomic).

Having both parts of (156) divided by $ds^2{=}c^2d\tau^2\left(1{-}\frac{\mathrm{v}^2}{c^2}\right)$ we obtain quadratic equation in respect to $\frac{dt}{d\tau}$ (31), which has two solutions similar to (32). From this formula we see that coordinate time increases $\frac{dt}{d\tau}{>}0$, stops $\frac{dt}{d\tau}{=}0$ and decreases $\frac{dt}{d\tau}{<}0$ under following conditions

$$\frac{dt}{d\tau} > 0 \qquad \text{if} \quad v_i\mathrm{v}^i > \pm c^2, \tag{157}$$

$$\frac{dt}{d\tau} = 0 \qquad \text{if} \quad v_i\mathrm{v}^i = \pm c^2, \tag{158}$$

$$\frac{dt}{d\tau} < 0 \qquad \text{if} \quad v_i\mathrm{v}^i < \pm c^2. \tag{159}$$

As known, regular real particles' velocities are lower than the speed of light. Therefore the condition of coordinate time stop $v_i\mathrm{v}^i{=}{\pm}c^2$ (158) can not be true in the world of substance, but is not impossible for other states of matter (for light-like matter, for instance).

Coordinate time increases $\frac{dt}{d\tau}{>}0$ (157) at $v_i\mathrm{v}^i{>}{\pm}c^2$. In a regular laboratory rotational velocities are also below the speed of light. Hence in regular laboratory conditions $v_i\mathrm{v}^i{>}{-}c^2$ (the angle α between the rotation velocity and the observable velocity is within the limits $-\frac{\pi}{2}{<}\alpha{<}\frac{\pi}{2}$). In this case flow of coordinate time is direct, i. e. from past into future.

Coordinate time decreases $\frac{dt}{d\tau}{<}0$ at $v_i\mathrm{v}^i{<}{\pm}c^2$.

Until now we have looked at flow of coordinate time t only. Now we are going to analyze possible directions of physical observable time τ, which depends upon the sign of derivative $\frac{d\tau}{dt}$. We obtain this value by dividing the formula for $d\tau$ (155) by dt

$$\frac{d\tau}{dt} = 1 - \frac{1}{c^2}\left(w + v_i u^i\right). \tag{160}$$

By definition, regular observer's clock always counts positive intervals of time irrespective of in what direction clock's hands rotate. Therefore in a regular laboratory on Earth physical observable time may increase or stop, but it never decreases. Nevertheless decrease of observable time $\frac{d\tau}{dt}{<}0$ is possible in certain circumstances.

From (160) we see that observable time increases $\frac{d\tau}{dt}{>}0$, stops $\frac{d\tau}{dt}{=}0$ or decreases $\frac{d\tau}{dt}{<}0$ under the following conditions

$$\frac{d\tau}{dt} > 0 \qquad \text{if} \quad w + v_i u^i < c^2, \tag{161}$$

$$\frac{d\tau}{dt} = 0 \qquad \text{if} \quad w + v_i u^i = c^2, \tag{162}$$

$$\frac{d\tau}{dt} < 0 \qquad \text{if} \quad w + v_i u^i > c^2. \tag{163}$$

Evidently, the condition of observable time stop $w{+}v_iu^i{=}c^2$ is also the condition of degeneration of space-time (56). In a partial case, when rotation of space is absent, observable time stops in collapse $w{=}c^2$.

Generally zero-space is described by whole spectrum of physical conditions represented as $w{+}v_iu^i{=}c^2$. Black holes ($w{=}c^2$) is only a partial case of such conditions in absence of space's rotation $v_i{=}0$. In other words, the *mirror membrane* between the world with direct flow of time and the world with reverse flow of time (the mirror world) are not black holes in zero-space alone, but the zero-space in general.

So what is flow of coordinate time t and what is flow of physical observable time τ?

In the function of coordinate time $\frac{dt}{d\tau}$ we assume that observer's measured time τ is the standard, in respect to which time coordinate t is defined. Here we are linked to the observer themselves and from their point of view we define where do they travel (into past, into future or rests). That is, the function of coordinate time $\frac{dt}{d\tau}$ defines *observable motion* of the observer along time axis $x^0{=}ct$ from their own viewpoint.

In the function of observable time $\frac{d\tau}{dt}$ change of observer's temporal coordinate t is the standard. That is, observer's measured time τ is defined in respect to motion of the whole observer's spatial section along the axis of time, which occurs evenly at the speed of light. Therefore function of observable time $\frac{d\tau}{dt}$ gives a view of the observer from aside, showing their *true motion* in respect to time axis.

In other words, the function of coordinate time $\frac{dt}{d\tau}$ shows the membrane between the world with direct flow of time and that with the reverse flow of time from "inside", from viewpoint of the observer who travels into future or into past. The function of observable time $\frac{d\tau}{dt}$ gives a look at the membrane from "outside", from viewpoint of space-time itself. This means that it is the function of observable time $\frac{d\tau}{dt}$ that shows the *true structure* of space-time membrane between the worlds with direct and reverse flow of time.

9 Basic introduction into the mirror world

To obtain a more detailed view of space-time membranes we are going to use *local geodesic frame of reference*. Fundamental metric tensor within infinitesimal vicinities of any point of such frame is

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \frac{1}{2}\left(\frac{\partial^2 \tilde{g}_{\mu\nu}}{\partial \tilde{x}^\rho \partial \tilde{x}^\sigma}\right)(\tilde{x}^\rho - x^\rho)(\tilde{x}^\sigma - x^\sigma) + \dots \ , \tag{164}$$

i. e. values of its components in the vicinities of a point is different from the those at this point itself are only different by figures of 2nd order of smallness and less which can be neglected. Therefore at any point of local geodesic frame of reference fundamental metric tensor (up within the 2nd order of smallness figures) is a constant, while the first derivatives of the metric, i. e. Christoffel symbols, are zeroes [10].

Evidently within infinitesimal vicinities of any point of Riemannian space a local geodesic frame of reference can be set. Subsequently at any point of local geodesic frame of reference a tangential flat space can be set so that local geodesic frame of reference of the Riemannian space is global geodesic one for that flat space. Because in a flat space metric tensor is constant, in the vicinities of a point of Riemannian space the values $\tilde{g}_{\mu\nu}$ converge to values of that tensor $g_{\mu\nu}$ in tangential flat space. That means that in a tangential flat space we can build a system of basic vectors $\vec{e}_{(\alpha)}$ tangential to curved coordinate lines of the Riemannian space. Because coordinate lines of Riemannian space are generally curved and in non-holonomic space are not even orthogonal to each other, lengths of basic vectors are sometimes substantially different from unit length.

Let $d\vec{r}$ be a four-dimensional vector of infinitesimal displacement $d\vec{r}{=}(dx^0, dx^1, dx^2, dx^3)$. Then $d\vec{r}{=}\vec{e}_{(\alpha)}dx^\alpha$, where the components are

$$\begin{aligned} \vec{e}_{(0)} &= (e^0_{(0)}, 0, 0, 0), & \vec{e}_{(1)} &= (0, e^1_{(1)}, 0, 0), \\ \vec{e}_{(2)} &= (0, 0, e^2_{(2)}, 0), & \vec{e}_{(3)} &= (0, 0, 0, e^3_{(3)}). \end{aligned} \tag{165}$$

Scalar product of vector $d\vec{r}$ with itself gives $d\vec{r}d\vec{r}{=}ds^2$, i. e. the square of four-dimensional interval. On the other hand $ds^2{=}g_{\alpha\beta}dx^\alpha dx^\beta$. Hence

$$g_{\alpha\beta} = \vec{e}_{(\alpha)}\vec{e}_{(\beta)} = e_{(\alpha)}e_{(\beta)}\cos(x^\alpha; x^\beta), \tag{166}$$

which facilitates better understanding of geometric structure of different areas within Riemannian space and even beyond. According to the formula

$$g_{00} = e^2_{(0)}\,, \tag{167}$$

and on the other hand $\sqrt{g_{00}}{=}1{-}\frac{w}{c^2}$. Hence length of temporal basic vector $\vec{e}_{(0)}$ tangential to coordinate line of time $x^0{=}ct$ is

$$e_{(0)} = \sqrt{g_{00}} = 1 - \frac{w}{c^2} \tag{168}$$

and is the lesser than one the greater is the gravitational potential w. In case of collapse ($w{=}c^2$) the length of temporal basic vector $\vec{e}_{(0)}$ becomes zero.

According to (166) the value g_{0i} is

$$g_{0i} = e_{(0)}e_{(i)}\cos(x^0; x^i), \tag{169}$$

on the other hand $g_{0i}{=}{-}\frac{1}{c}v_i\left(1{-}\frac{w}{c^2}\right){=}{-}\frac{1}{c}v_i e_{(0)}$. Hence

$$v_i = -c e_{(i)}\cos(x^0; x^i). \tag{170}$$

Then according to general formula (166)

$$g_{ik} = e_{(i)}e_{(k)}\cos(x^i; x^k), \tag{171}$$

we obtain that observable metric tensor $h_{ik}{=}{-}g_{ik}{+}\frac{1}{c^2}v_i v_k$ takes the form

$$h_{ik} = e_{(i)}e_{(k)}\left[\cos(x^0; x^i)\cos(x^0; x^k) - \cos(x^i; x^k)\right]. \tag{172}$$

From (170) we see that geometrically v_i is a projection (scalar product) of spatial basic vector $\vec{e}_{(i)}$ onto temporal basic vector $\vec{e}_{(0)}$, multiplied by speed of light. If spatial sections are everywhere orthogonal to lines of time (holonomic space), then $\cos(x^0; x^i){=}0$ and $v_i{=}0$. In non-holonomic space spatial sections are not orthogonal to lines of time and $\cos(x^0; x^i){\neq}0$. Generally $|\cos(x^0; x^i)|{\leq}1$ hence velocity of space rotation v_i (170) can not exceed speed of light.

If $\cos(x^0; x^i){=}{\pm}1$, then velocity of space rotation is

$$v_i = \mp c e_{(i)}\,, \tag{173}$$

and temporal basic vector $\vec{e}_{(0)}$ coincides with spatial basic vectors $\vec{e}_{(i)}$ (time "falls" into space). At $\cos(x^0; x^i){=}{+}1$ temporal basic vector is co-directed with the spatial ones $\vec{e}_{(0)}{\uparrow\uparrow}\vec{e}_{(i)}$. In case $\cos(x^0; x^i){=}{-}1$ temporal and spatial basic vectors are oppositely directed $\vec{e}_{(0)}{\uparrow\downarrow}\vec{e}_{(i)}$.

Let us have a closer look at the condition $\cos(x^0; x^i){=}{\pm}1$. If any spatial basic vector is co-directed (or oppositely directed) to the temporal basic vector the space is degenerated. Maximum degeneration occurs when all three vectors $\vec{e}_{(i)}$ coincide with each other and with the temporal basic vector $\vec{e}_{(0)}$.

The condition of stop of coordinate time $v_i\mathrm{v}^i{=}{\pm}c^2$ presented through the basic vectors is

$$e_{(i)}v^i\cos(x^0; x^i) = \mp c \tag{174}$$

and becomes true when $e_{(i)}{=}1$, $\mathrm{v}{=}c$ and $\cos(x^0; x^i){=}{\pm}1$. In this case if the velocity of rotation reaches the speed of light the angle between the time line and the spatial lines becomes either zero or π depending upon the rotation direction.

Let us illustrate this with a few examples.

1. Space does not rotate, i. e. is holonomic

In this case $v_i{=}0$ and spatial sections are everywhere orthogonal to lines of time and the angle between them is $\alpha{=}\frac{\pi}{2}$. Hence in absence of space rotation temporal basic vector $\vec{e}_{(0)}$ is orthogonal to all spatial basic vectors $\vec{e}_{(i)}$. That means that all clocks can be synchronized and will display the same time (synchronization of clocks at different points in space does not depend upon ways of synchronization). Velocity of space rotation $v_i{=}{-}ce_{(i)}\cos\alpha{=}0$. At $v_i{=}0$

$$d\tau = \left(1-\frac{w}{c^2}\right)cdt\,, \qquad\qquad h_{ik} = -g_{ik}\,, \tag{175}$$

and metric of space-time $ds^2{=}c^2d\tau^2{-}d\sigma^2$ becomes

$$ds^2 = \left(1-\frac{w}{c^2}\right)^2 c^2dt^2 + g_{ik}dx^idx^k, \tag{176}$$

i. e. pace of observable time depends only upon gravitational potential w. Two options are possible here. (a) Gravitational inertial force $F_i{=}0$ and space's rotation $v_i{=}0$. Then according to definitions of F_i and v_i (see Section 2) we have $\sqrt{g_{00}}{=}1{-}\frac{w}{c^2}{=}1$ and $g_{0i}{=}{-}\frac{1}{c}\sqrt{g_{00}}\,v_i{=}0$. Equality to zero of gravitational potential w means, in particular, that it does not depend upon three-dimensional coordinates. In this case motion of observer across space where no rotation is present leaves pace of different clocks the same (global synchronization is preserved with time). (b) If $F_i{\neq}0$ and $v_i{=}0$ then in the formula for F_i (19) the derivative $\frac{\partial w}{\partial x^i}{\neq}0$. That means that gravitational potential depends upon three-dimensional coordinates, i. e. pace of time is different at different points of space. Hence at $F_i{\neq}0$ synchronization of clocks at different point of space where no rotation is present does not preserve with time.

In a space where no rotation is present collapsed matter may exist (black holes, $w{=}c^2$) only if $F_i{\neq}0$. If $F_i{=}0$ then according to definition of F_i (19) in space where no rotation is present $w{=}0$ and gravitational collapse is not possible.

2. Space rotates at sub-light velocity

Consequently in this case spatial sections are not orthogonal to lines of time $v_i{=}{-}ce_{(i)}\cos\alpha{\neq}0$. Because $-1{\leq}\cos\alpha{\leq}{+}1$, then $-c{\leq}v_i{\leq}{+}c$. Hence $v_i{>}0$ at $\cos\alpha{>}0$ and $v_i{<}0$ at $\cos\alpha{<}0$.

3. Space rotates at light velocity (first case)

The lesser is α the greater is v_i. In the ultimate case when $\alpha{=}0$ velocity of space rotation $v_i{=}{-}c$. Consequently spatial basic vectors $\vec{e}_{(i)}$ coincide with temporal basic vector $\vec{e}_{(0)}$ (space coincides with time).

4. Space rotates at light velocity (second case)

If $\alpha{=}\pi$, then $v_i{=}{+}c$ and temporal basic vector $\vec{e}_{(0)}$ also coincides with spatial basic vectors $\vec{e}_{(i)}$ but is oppositely directed. The case may be interpreted as space that coincides with "anti-time" which flows from future into past.

10 Motion of particles as a result of space's motion

10.1 Problem statement

Having substituted gravitational potential w and velocity of space rotation v_i into definition of interval of observable time $d\tau$ (13), we obtain the expression (13) as

$$\left(1+\frac{1}{c^2}v_i\mathrm{v}^i\right)d\tau = \left(1-\frac{w}{c^2}\right)dt\,. \tag{177}$$

From here we see that significant difference between $d\tau$ and dt may result from either strong gravitational field or velocities comparable to speed of light. Hence in everyday life the difference between observable time τ and coordinate time t is not great.

Physical observable time coincides with the time coordinate $dt{=}d\tau$ only under the condition

$$w = -v_i \mathrm{v}^i. \tag{178}$$

Actually such condition implies that attraction of particle by the central body is fully compensated by rotation of space and rotation of the particle itself. That is (178) is mathematical interpretation of the *weightlessness condition.* Having gravitational potential substituted according to Newton' formula we obtain

$$\frac{GM}{r} = v_i \mathrm{v}^i. \tag{179}$$

If orbital velocity of particle equals to velocity of rotation of the central body in this orbit the condition of weightlessness for the particle becomes

$$\frac{GM}{r} = v^2, \tag{180}$$

i. e. the more distant is the orbit from the central body, the lesser is the velocity of a satellite in this orbit.

Is this statement confirmed by experimental data? The Table in the below gives orbital velocities of the Moon and the planets s measured in astronomical observations as well as those calculated from the condition of weightlessness.

Planet	**Orbital velocity, km/s**	
	Measured	**Calculated**
Mercury	47.9	47.9
Venus	35.0	35.0
Earth	29.8	29.8
Mars	24.1	24.1
Jupiter	13.1	13.1
Saturn	9.6	9.6
Uran	6.8	6.8
Neptune	5.4	5.4
Pluto	4.7	4.7
Moon	1.0	1.0

From the Table we see that our condition of weightlessness is true for any satellite that orbits a central body. Noteworthy, the condition is true when orbital velocity of a planet equals to (or is close to) the velocity of rotation of the space of the central body in this orbit (180). That means that rotating space of the central body *carries* any and all bodies around making them rotate.

If the space of the central body rotated like a solid body i. e. without any deformation its angular velocity would be constant $\omega{=}const$ while orbital velocities $\mathrm{v}{=}\omega r$ of the carried satellites would grow along with radiuses of their orbits. But as we have just seen from the example of the Solar system planets, rotation velocity declines along with distance from the Sun. That means that in reality space of the central body (space of reference) does not rotate like a solid body, but rather like a viscous and deformable medium, where layers distant from the center do not rotate as far as those closer to the center. As a result space of the central body is *twisted* and the profile of orbital velocities simply repeats the structure of twisted space.

Hence we see that orbital motion of particles in gravitational field results from *rotation of the space of the attracting body itself.*

What are possible consequences for mathematical theory of motion of particles of the conclusions we have just made? We are going to see that in the below.

Let us assume a metric space. Evidently, motion of the space allows to match any of its points to vector of motion of such point Q^α. It is also evident that all points of the space will experience the same motion as the space itself. Hence Q^α can be regarded the vector of motion of the space itself (in a given point). As a result we obtain a vector field that describes motion of the whole space.

Of course if length of vector Q^α is constant the space will move so that its metric will stay the same too. Hence if in such space vector of motion Q^α is set in a given point then metric of the space can be found processing from motion of the point (along with motion of the space)

A way to solve the problem was paved in the late 19th century by S. Lie [12]. He obtained equations of derivative from space metric $g_{\alpha\beta}$ to trajectory of motion of vector Q^{α}, which contained components of Q^{α} as fixed coefficients. The number of the equations as qual to the number of components of the metric. Hence having vector Q^{α} fixed, i. e. having motion of the space set, we can solve the equations to find all components of metric $g_{\alpha\beta}$ proceeding from the components Q^{α}.

Later Van Danzig suggested to call such derivative of metric *Lie derivative*.

Now we are going to look at a partial case of motion of space which leaves its metric constant. The case was studied by W. Killing [13]. Evidently such motion will make Lie derivative equal to zero (*Killing equations*). Hence if motion of space leaves its metric the same and we know vector Q^{α} for any of its points (i. e. motion of the space at this point is set), motion of the point can be used to obtain metric of the space from Killing equations.

On the other hand motion of particles is described by dynamic equations of motion. To the contrary, these equations leave metric of space fixed and the problem here is to find dynamic vector of motion of particle Q^{α}. Fixed metric in dynamic equations of motion makes Christoffel symbols, which are functions of metric components $g_{\alpha\beta}$, appear in the equations as fixed coefficients. Hence as soon as particular metric of space is set we can use dynamic equations of motion to obtain vector Q^{α} for the particle in such space.

Therefore we arrive to the following. Because $g_{\alpha\beta}$ is a symmetric tensor ($g_{\alpha\beta}=g_{\beta\alpha}$) only 10 its components out of 16 have different values. In Killing equations (10 equations) vector of motion of a point in space is fixed, while components of the metric are unknown (10 unknowns). Dynamic equations of motion of free particles (4 equations), to the contrary, leave metric fixed, but components of vector of motion of particles (4 components) are unknown. Then as soon as we look at free motion of particle as motion of the point in space carried by motion of the space itself, we can make a system of 10 Killing equations (equations of motion of space) and of 4 dynamic equations of motion of particle. The system of 14 equations will have 14 unknowns, 10 out of which are unknown components of metric and 4 are unknown components of dynamic vector of particle. Hence having this system solved we obtain motion of particle in space and metric of the space at the same time.

In particular, solving the system we can find motions of particles which result from motion of space itself. For such type of motion knowledge of motion of a certain particle can produce metric of space.

For instance, having Killing equations and dynamic equations of motion solved for a satellite (or a planet) we can use its motion to find metric of the space of the central body.

In the next Section we are going to obtain Killing equations in chronometrically invariant form.

10.2 Equations of motion and Killing equations

Let us assume a space (not necessarily a metric one) that moves. Evidently vector of motion of any point of the space Q^{α} is vector of motion of the space itself at this point. Motion of space is described by *Lie derivative*

$$\underset{L}{\delta}\, g_{\alpha\beta} = Q^{\alpha}\frac{\partial g_{\alpha\beta}}{\partial x^{\sigma}} + g_{\alpha\sigma}\frac{\partial Q^{\sigma}}{\partial x^{\beta}} + g_{\beta\sigma}\frac{\partial Q^{\sigma}}{\partial x^{\alpha}}\,, \tag{181}$$

which is derivative of the metric of the space to direction of motion of vector Q^{α} (direction of motion of the space itself).

We are now looking to the picture as follows. We assume a point in space. If space moves the point will be subjected to action of carrying vector Q^{α} which is vector of motion of the space itself. For the point itself the space will rest and only “the wind” produced by motion of space as vector Q^{α} will disclose motion of the whole space.

Generally Lie derivative is not zero, that is motion of space alters its metric. But in Riemannian space metric is fixed by definition and length of vector being transferred parallel to itself is constant. That means that parallel transfer of vector across “non-smooth” structure of Riemannian space will alter the vector along with configuration of the space. As a result Lie derivative of metric in Riemannian space will be zero

$$\underset{L}{\delta}\, g_{\alpha\beta} = 0\,. \tag{182}$$

Lie equations in Riemannian space were first studied by Killing and are referred to as Killing equations. Later A. Z. Petrov showed [14] that Killing equations for any point are the necessary and sufficient condition for the motion of the point to be motion of Riemannian space itself. In other words if a point is carried by motion of Riemannian space and moves along, Killing equations must be true for that point.

Evidently to obtain components of metric out of Killing equations we need to set a particular vector of motion of a point Q^α. Then we will have 10 Killing equations vs. 10 unknown metric components and will able to solve the system.

Generally there might be different kinds of motion of Riemannian space. We will set vector of motion Q^α as to fit the needs of our problem.

There exists *free* (*geodesic*) *motion* in which a point moves along geodesic (the shortest) trajectory. We assume that any point of Riemannian space carried by motion of the space itself moves along geodesic trajectory. Hence motion of the entire Riemannian space will be geodesic as well. Then we can match motion of a point carried by motion of space to motion of a free particle.

We will call a motion *geodesic motion of space* if free motion of particles results from their carrying by moving space.

Let us look at a system of dynamic equations of motion of free particles and Killing equations

$$\left.\begin{array}{l} \dfrac{DQ^\alpha}{d\rho}=0 \\ \underset{L}{\delta}\, g_{\alpha\beta}=0 \end{array}\right\}, \tag{183}$$

where Q^α stands for dynamic vector of motion of particle, ρ stands for derivation parameter to trajectory of motion while Lie derivative can be expressed through *Lie differential* as

$$\underset{L}{\delta}\, g_{\alpha\beta}=\frac{\underset{L}{D}\, g_{\alpha\beta}}{d\rho}\,. \tag{184}$$

Actually this system of equations means that motion of free particle is geodesic one (equations of motion of particles) and at the same time results from carrying of particle by geodesic motion of the space (Killing equations). The system solves as a set of components of the dynamic vector Q^α and components of metric $g_{\alpha\beta}$ for which geodesic motion of particles results from geodesic motion of the space itself.

To solve the problem correctly we need to present Killing equations in chronometrically invariant form thus presenting them through physical properties of space (standards). It is especially interesting to know which of physical standards result from motion of space itself.

According to theory of chronometric invariants (12) physical observable values are projection of Killing equation on time (1 component), mixed projection (3 components) and spatial projection (6 components)

$$\frac{\underset{L}{\delta}\, g_{00}}{g_{00}}=0\,, \tag{185}$$

$$\frac{\underset{L}{\delta}\, g_0^i}{\sqrt{g_{00}}}=\frac{g^{i\alpha}\,\underset{L}{\delta}\, g_{0\alpha}}{\sqrt{g_{00}}}=0\,, \tag{186}$$

$$\underset{L}{\delta}\, g^{ik}=g^{i\alpha}g^{k\beta}\,\underset{L}{\delta}\, g_{\alpha\beta}=0\,. \tag{187}$$

Here we are looking at motion of space and particles from the viewpoint of a regular sub-light-speed observer.

Having presented derivatives of metric in Lie derivative through chronometric invariant operators and having substituted short notation of observable components of dynamic vector of particle Q^α as $\varphi=\dfrac{Q_0}{\sqrt{g_{00}}}$ and $q^i=Q^i$, we arrive to *chronometrically invariant Killing equations*

$$\frac{{}^*\partial\varphi}{\partial t}-\frac{1}{c}F_iq^i=0\,, \tag{188}$$

$$\frac{1}{c}\frac{{}^*\partial q^i}{\partial t} - h^{im}\frac{{}^*\partial\varphi}{\partial x^m} - \frac{\varphi}{c^2}F^i + \frac{2}{c}A_{k\cdot}^{\cdot i}q^k = 0\,, \tag{189}$$

$$\frac{2\varphi}{c}D^{ik} + h^{im}h^{kn}q^l\frac{{}^*\partial h_{mn}}{\partial x^l} + h^{im}\frac{{}^*\partial q^k}{\partial x^m} + h^{km}\frac{{}^*\partial q^i}{\partial x^m} = 0\,. \tag{190}$$

If vector Q^α at the same time complies chronometrically invariant Killing equations and chronometrically invariant dynamic equations of motion of particle, then such particles moves being carried by geodesic motion of space.

Joint solution of the equations in general form is problematic and so we will limit ourselves to a single partial case, which is still of great importance. Let dynamic vector of motion of space Q^α be dynamic vector of motion of mass-bearing particles

$$Q^\alpha = m_0\frac{dx^\alpha}{ds} = \frac{m}{c}\frac{dx^\alpha}{d\tau}\,, \tag{191}$$

and let observer accompany the particle (v^i=0). Then

$$\varphi = m_0 = const, \qquad q^i = \frac{m}{c}\mathrm{v}^i, \tag{192}$$

and Killing equations (188–190) are simplified to

$$F^i = 0\,, \qquad\qquad D^{ik} = 0\,. \tag{193}$$

According to (20) D^{ik}=0 means stationary state of observable metric h^{ik}=*const*. The condition F^i=0 means that for the following equalities to become only true by transformation of time coordinate

$$g_{00} = 1\,, \qquad\qquad \frac{\partial g_{0i}}{\partial t} = 0\,. \tag{194}$$

Besides, F^i and A_{ik} are related through Zelmanov identity [10]

$$\frac{1}{2}\left(\frac{{}^*\partial F_k}{\partial x^i} - \frac{{}^*\partial F_i}{\partial x^k}\right) + \frac{{}^*\partial A_{ik}}{\partial t} = 0\,, \tag{195}$$

from which we see that F^i=0 means

$$\frac{{}^*\partial A_{ik}}{\partial t} = 0\,, \tag{196}$$

i. e. rotation of space of reference is *stationary*.

Further, as seen from Killing equations (193) tensor of deformation velocities of space of reference is zero, hence *stationary rotation* does not alter structure of space. Equality to zero of gravitational inertial force in Killing equations means that from the viewpoint of carried particle (v^i=0) it weighs nothing and is not attracted to anything (weightlessness state). This does not contradict with the weightlessness condition w=$-v_i\mathrm{v}^i$ obtained earlier because from the viewpoint of carried particle gravitational potential of the body of reference w=0 and F^i=0 as well.

Hence if Q^α is a vector of motion of mass-bearing particles, then *geodesic motion of space* along this vector is *stationary rotation*.

As we see geodesic motion of mass-bearing particles is stationary rotation. And such stationary rotation results from carrying by the space of the gravitating body. But we know that the basic type of motion in the Universe is orbiting. Hence the basic motion in the Universe is a geodesic motion which results from carrying of objects by stationary (geodesic) rotation of spaces of their central bodies.

10.3 Calculating mass of the Galaxy

We will proceed from the fact that orbital motion of the Sun around the center of the Galaxy complies with the condition of weightlessness. If space of our Galaxy *fully* carries the Sun then the orbital velocity of the Sun v^i will be equal to velocity of rotation of the Galactic space v_i in the same orbit. Hence the condition of weightlessness for the Sun will be quite simple (180). After substituting distance from the center of our Galaxy to the Sun r=3·10^{22} cm and orbital velocity of the Sun 250 km/s we obtain the mass of our Galaxy

$$M = \frac{r v_i \mathrm{v}^i}{G} = \frac{r v^2}{G} = 2.8 \cdot 10^{44}\,\mathrm{g}. \tag{197}$$

According to astronomical data the mass of our Galaxy is $3{\cdot}10^{44}$ g (within the sphere of 15,000 parsec). The true mass of our Galaxy is a bit over the value. A small difference from the figure $2.8{\cdot}10^{44}$ g we have obtained can be explained by the fact that the Sun is *not fully* carried by rotation of the Galactic space. Therefore it seems reasonable to introduce so-called *coefficient of carrying*

$$k = \frac{\mathrm{v}}{v}\,, \tag{198}$$

and the condition of weightlessness becomes

$$\frac{GM}{r} = \frac{\mathrm{v}^2}{k}\,. \tag{199}$$

Having substituted the observed mass of the Galaxy $3{\cdot}10^{44}$ g into here, we obtain the coefficient of carrying of the Sun by rotation of the Galactic space (because we have substituted the least estimate $3{\cdot}10^{44}$ g the true value of the coefficient should be even lower)

$$k = \frac{r\mathrm{v}^2}{GM} = 0.94\,. \tag{200}$$

10.4 Resume

So what is the space if it bears gravitational potential w can be deformed and in rotation behaves like viscous media? Noteworthy, if we place a particle into the space, the moving space will carry it just like current in the ocean carries a tiny boat and a giant iceberg.

The answer is as follows: according to the results we have obtained in the above, the space of reference of a body and its gravitational field are the same thing. And physically speaking, points of the space of reference are particles of gravitational field of the body of reference.

If the space of reference does not rotate, a satellite will fall on the body of reference under action of gravitational force. But in presence of rotation the satellite will be under action of carrying force. The force will act like wind of oceanic stream to push the satellite forward, not allowing it to fall down and making it orbit the central body along with the rotating space (of course additional velocity given to the satellite will make it move faster than the rotating space).

11 Who is a super-light observer?

We can outline a few types of frames of reference which may exist in General Relativity space-time. Particles (including the observer themselves), which travel at sub-light speed ("inside" the light cone), bear real relativistic mass. In other words, the particles, the body of reference and the observer are in the state of matter commonly referred to as "substance". Therefore any observer whose frame of reference is described by such monad will be referred to as *sub-light speed* (*substantional*) *observer.*

Particles and the observer that travel at the speed of light (i. e. over the surface of light hypercone) bear m_0=0 but their relativistic mass (mass of motion) $m{\neq}0$. They are in light-like state of matter. Hence we will call an observer whose frame of reference is characterized by such monad a *light-like observer.*

Accordingly, we will call particles and the observer that travel at super-light speed *super-light* particles and observer. They are in the state of matter for which $m_0{\neq}0$ but their relativistic mass is imaginary.

It is intuitively clear who a sub-light speed observer is, the term requires no further explanations. Same more or less applies to light-like observer. From point of view of light-like observer the world around looks like colorful system of light waves. But who is a super-light observer? To understand this let us give an example.

Imagine a new supersonic jet plane to be commissioned into operation. All members of the commission are inborn blind. And so is the pilot. Thus we may assume that all information about the surrounding world the pilot and the members of the commission gain from sound, that is from

transversal waves in air. It is sound waves that build a picture that those people will perceive as their "real world".

Now the plane took off and began to accelerate. As long as its speed is less than the speed of sound, the blind members of the commission will match its "heard" position in the sky to the one we can see. But once the sound barrier is overcome, everything changes. Blind members of the commission will still perceive the speed of the plane equal to the speed of sound regardless to its real speed. For the speed of propagation of sound waves in the air will be the *maximum speed of propagation of information* while the real supersonic jet plane will be beyond their "real world" in the world of "imaginary objects" and all its properties will be imaginary too. The blind pilot will hear nothing as well. Not a single sound will reach him from the past reality and only local sounds from the cockpit (which also travels at the supersonic speed) will break the silence. Once the speed of sound is overcome, the blind pilot leaves the subsonic world for a new supersonic one. From his new viewpoint (supersonic frame of reference) the old subsonic fixed world tat contains the airport and the members of the commission will simply disappear to become an area of "imaginary values".

What is light? Transversal waves that run across a certain medium at a constant speed. We perceive the world around through sight, receiving light waves from other objects. It is waves of light that build our picture of the "true real world".

Now imagine a spaceship that accelerates faster and faster to eventually overcome the light barrier at still growing speed. From pure mathematical viewpoint this is quite possible in the space-time of General Relativity. For us the speed of the spaceship will be still equal to the speed of light whatever is its real speed. For us the speed of light will be the maximum speed of propagation of information and the real spaceship for us will stay in another "unreal" world of super-light speeds where all properties are imaginary. The same is true for the spaceship's pilot. From his viewpoint having the light barrier overcome brings him into a new super-light world that becomes his "true reality". And the old world of sub-light speeds is gone to the area of "imaginary reality".

12 World of black holes

We will call *black hole* (gravitational collapser) an area of space-time where the condition $g_{00}=0$ is true [10, 11]. Because according to the theory of chronometric invariants $\sqrt{g_{00}}=1-\frac{w}{c^2}$, the condition of collapse $g_{00}=0$ also implies $w=c^2$. We will look at black holes from outside, from viewpoint of a regular observer who stays above the surface of collapser.

We put down the formula for four-dimensional interval so that it contains an explicit ratio of w and c^2

$$ds^2 = \left(1-\frac{w}{c^2}\right)^2 c^2dt^2 - 2\left(1-\frac{w}{c^2}\right) v_i dx^i dt + g_{ik}dx^i dx^k. \tag{201}$$

Having substituted $w=c^2$ into here we obtain the metric on the surface of black hole

$$ds^2 = g_{ik}dx^i dx^k. \tag{202}$$

From here we see that the collapse in four-dimensional space-time can be correctly defined only if three-dimensional space does not rotate i. e. is holonomic.

As a matter of fact the denominator of velocity of space rotation

$$v_i = -c\frac{g_{0i}}{\sqrt{g_{00}}} = -c\frac{g_{0i}}{1-\dfrac{w}{c^2}} \tag{203}$$

turns zero in case of collapse ($w=c^2$) and the rotation velocity becomes infinite. To avoid this we assume $g_{0i}=0$. Then metric (201) becomes

$$ds^2 = \left(1-\frac{w}{c^2}\right)^2 c^2dt^2 + g_{ik}dx^i dx^k, \tag{204}$$

and the problem of peculiar state of space will be automatically removed. Proceeding from this the metric on the surface of black hole (202) is

$$ds^2 = -d\sigma^2 = -h_{ik}dx^i dx^k, \qquad h_{ik} = -g_{ik}\,. \tag{205}$$

From here we see that four-dimensional potential on the surface of gravitational collapser is space-like — elementary distance between two point on the surface of black hole is imaginary

$$ds = id\sigma = i\sqrt{h_{ik}dx^i dx^k}\,. \tag{206}$$

If ds=0 three-dimensional observable distance $d\sigma$ between two points on the surface of collapser also becomes zero. Because in absence of rotation of space the interval of observable space equals $d\tau{=}\sqrt{g_{00}}\,dt{=}\left(1{-}\frac{w}{c^2}\right)dt$, on the surface of collapser $d\tau$=0 (observable time stops).

Now we are going to look at collapse in different areas of four-dimensional space-time.

1. Collapse in sub-light area

Within this area $ds^2{>}0$. This is the habitat of regular real particles that travel at sub-light speeds. Hence collapser in this area consists of collapsed substance (*substantional black hole*). On the surface of such collapser metric is space-like: here $ds^2{<}0$ and particles bear imaginary relativistic masses. Of course metric on the surface of real black hole is not degenerated.

2. Collapse in light-like area

Within this area ds^2=0. This is isotropic space of light-like (massless) particles. Collapser in this area is made of collapsed light-like matter (*light-like black hole*). Metric (205) on its surface is $d\sigma^2{=}{-}g_{ik}dx^i dx^k{=}0$. This can be true provided that: (a) surface of light-like collapser shrinks to a point (all dx^i=0), or (b) three-dimensional spatial metric is degenerated $g_{(3D)}{=}\det\|g_{ik}\|{=}0$ (because four-dimensional metric is degenerated too light-like collapser is zero-space in this case).

3. Collapse in degenerated space-time (zero-space)

Degenerated matter of zero-space can collapse too. We will call such collapsers *degenerated black holes*. As a matter of fact, from the condition of degeneration of space-time

$$w + v_i u^i = c^2, \qquad g_{ik}dx^i dx^k = \left(1 - \frac{w}{c^2}\right)^2 c^2 dt^2, \tag{207}$$

we see that in case of collapse ($w{=}c^2$)

$$v_i u^i = 0\,, \qquad g_{ik}dx^i dx^k = 0\,. \tag{208}$$

Hence collapse in zero-space also occurs in absence of rotation (v_i=0). And because conditions (208) are true at the same time the surface of degenerated black hole is shrunk into a point.

13 Geometric structure of zero-space

Regular real observer perceives zero-space as an area defined by the observableconditions $d\tau$=0 and $d\sigma^2{=}h_{ik}dx^i dx^k{=}0$.

Physical sense of the first condition $d\tau$=0 is that real observer perceives any two events in zero-space as simultaneous, whatever distant from them they are. Such way of instantaneous spread of information is referred to as *long-range action*.

The second condition $d\sigma^2$=0 means absence of observable distance between the event and the observer. Such "superposition" of observer and observed object is only possible if we assume that our regular four-dimensional pseudo-Riemannian space is "stuffed" with zero-space.

Let us now turn to mathematical interpretation of degeneration conditions.

The value $cd\tau$ is a projection of four-dimensional interval dx^α onto time $cd\tau{=}b_\alpha dx^\alpha$. Monad vector of the observer b^α by definition is not zero and dx^α are not zeroes too. Then $d\tau$=0 is true at $d\sigma^2$=0 only if four-dimensional metric $d\sigma^2{=}c^2d\tau^2{-}d\sigma^2{=}g_{\alpha\beta}dx^\alpha dx^\beta$ is degenerated, i. e. determinant of fundamental metric tensor is zero

$$g = \det\|g_{\alpha\beta}\| = 0\,. \tag{209}$$

Similarly the condition $d\sigma^2{=}h_{ik}dx^i dx^k{=}0$ means that in that area three-dimensional observable metric is degenerated too

$$h = \det\|h_{ik}\| = 0\,. \tag{210}$$

Having the conditions of degeneration of space-time $w{+}v_iu^i{=}c^2$ and $g_{ik}dx^idx^k{=}\left(1{-}\frac{w}{c^2}\right)^2c^2dt^2$ substituted into ds^2 we obtain formula for the metric of zero-space

$$ds^2 = \left(1 - \frac{w}{c^2}\right)^2 c^2dt^2 - g_{ik}dx^idx^k = 0\,. \tag{211}$$

Hence inside zero-space three-dimensional space is holonomic while rotation is present in the temporal component of its metric

$$\left(1 - \frac{w}{c^2}\right)^2 c^2dt^2 = \left(\frac{v_iu^i}{c^2}\right)^2 c^2dt^2. \tag{212}$$

If $w{=}c^2$ (gravitational collapse) metric of zero-space (211) becomes

$$ds^2 = -g_{ik}dx^idx^k = 0\,, \tag{213}$$

i. e. becomes purely spatial. And three-dimensional space becomes degenerated too

$$g_{(3D)} = \det\|g_{ik}\| = 0\,. \tag{214}$$

Here the condition $g_{(3D)}{=}0$ is obtained from the fact that quadratic form $g_{ik}dx^idx^k$ is sign-definite and can only become zero provided the determinant of metric tensor g_{ik} equals to zero.

Because in zero-space $w{+}v_iu^i{=}c^2$ in case of gravitational collapse the condition $v_iu^i{=}0$ also becomes true.

The value $v_iu^i{=}vu\cos(v_i;u^i)$, which is scalar product of velocity of space rotation and coordinate velocity of particle will be referred to as *spirality* of zero-particle.

If $v_iu^i{>}0$ then the angle between v_i and u^i is within the range of $\frac{3\pi}{2}{<}\alpha{<}\frac{\pi}{2}$. Because the second condition of degeneration of space-time $g_{ik}u^iu^k{=}\left(1{-}\frac{w}{c^2}\right)^2$ implies that $u{=}c\left(1{-}\frac{w}{c^2}\right)$ then potential $w{<}c^2$ (regular gravitational field).

If $v_iu^i{<}0$ then α is within $\frac{\pi}{2}{<}\alpha{<}\frac{3\pi}{2}$ and $w{>}c^2$ (superstrong gravitational field).

The condition $v_iu^i{=}0$ is only true when $\alpha{=}\frac{\pi}{2};\frac{3\pi}{2}$ or if $w{=}c^2$ (gravitational collapse).

Hence spirality of zero-particle is zero if either velocity of particle is orthogonal to velocity of space rotation or gravitational collapse occurs (because in this case module of coordinate velocity of particle equals zero $u{=}0$).

Because $w{=}c^2(1{-}e_{(0)})$ and $v_i{=}{-}ce_{(i)}\cos(x^0;x^i)$ then condition of space degeneration $w{+}v_iu^i{=}c^2$ becomes

$$ce_{(0)} = -e_{(i)}u^i\cos(x^0;x^i). \tag{215}$$

Dimension of space is defined by the number of linearly independent basic vectors. In our formula (215), which is basic notation of the condition $w{+}v_iu^i{=}c^2$, temporal basic vector is linearly dependent from all spatial basic vectors. This means degeneration of space-time. Hence formula (215) can be regarded the geometric condition of degeneration.

In case of gravitational collapse ($w{=}c^2$) the length of temporal basic vector $e_{(0)}{=}1{-}\frac{w}{c^2}$ becomes zero. In absence of gravitational field ($w{=}0$, or in weak gravitational field $w{\to}0$) the value $e_{(0)}$ takes its maximum length equal to one. In intermediate cases $e_{(0)}$ becomes shorter as gravitational field becomes stronger.

As known in any point of four-dimensional space there exists so-called *isotropic cone* — a hypersurface which metric is

$$g_{\alpha\beta}dx^\alpha dx^\beta = 0\,. \tag{216}$$

Geometrically this is an area of space-time that hosts light-like particles. Because the square of interval in this area is zero, all directions inside are equal, i. e. are isotropic. Hence the area that hosts light-like particles is commonly referred to as *isotropic cone* or the *light cone*.

Because in zero-space metric is also equal to zero (211) an isotropic cone can be set in any of its points. But such will be described by a somewhat different equation

$$\left(1 - \frac{w}{c^2}\right)^2 c^2dt^2 - g_{ik}dx^idx^k = 0\,. \tag{217}$$

The difference between this isotropic cone and the light cone is that it satisfies the condition

$$1-\frac{w}{c^2}=\frac{v_i u^i}{c^2}\,, \tag{218}$$

which is only typical for degenerated space-time (zero-space). Hence we will call it *degenerated isotropic cone*. And because the specific term (218) is the direct function of space rotation, degenerated isotropic cone is a cone of rotation.

Under gravitational collapse ($w{=}c^2$) the first term in (217) becomes zero (the point of stop of coordinate time), while the remaining second term $g_{ik}dx^i dx^k{=}0$ describes three-dimensional degenerated hypersurface. But if $w{=}0$ then $v_i u^i{=}0$ and the equation of degenerated isotropic cone (217) becomes

$$c^2dt^2-g_{ik}dx^i dx^k=0\,, \tag{219}$$

i. e. coordinate time flows evenly.

The greater is gravitational potential w the "heavier" is degenerated cone and the closer it is to the spatial section. In the ultimate case when $w{=}c^2$ degenerated cone under action of its own "super-weight" becomes flattened over three-dimensional space (collapses). The "lightest" cone $w{=}0$ is the most distant one from spatial section.

Hence black hole in zero-space is similar to zero-space observed by regular real observer. In other words zero-space for us is degenerated state of regular space-time, while for zero-observer black hole is degenerated state of zero-space. That means that isotropic light cone contains degenerated isotropic cone of zero-space, which in its turn contains collapsed degenerated isotropic cone of the space inside black holes. This is illustration of fractal structure of the world presented here as a system of isotropic cones found inside each other.

14 Conclusions

Now we can build the general picture of motion of particles in four-dimensional space-time. Mass-bearing particles with positive relativistic masses $m{>}0$ inhabit our world with direct flow of time ("inner" part of the light cone) and move from past into future in respect to a regular observer. Particles with negative relativistic masses $m{<}0$ inhabit the mirror world and move from future into past in respect to us. In the mirror world time has reverse flow in respect to that in our world.

Inside the "walls" of the cone the condition $c^2d\tau^2{=}d\sigma^2$ is true, i. e. the temporal and the spatial projections of four-dimensional coordinates are equal, while the space-time interval is degenerated $ds^2{=}c^2d\tau^2{-}d\sigma^2{=}0$. This is the habitat of light-like (massless) particles. Light-like particles of our world with positive own frequencies $\omega{>}0$ move from pat into future. In the mirror world light-like particles with negative own frequencies $\omega{<}0$ move from future into past in respect to us.

Inside the membrane, which separates our world from the mirror world, a more strict condition is true $c^2d\tau^2{=}d\sigma^2{=}0$, i. e. observable time is degenerated $d\tau{=}0$ and observable three-dimensional metric is degenerated too $d\sigma^2{=}h_{ik}dx^i dx^k{=}0$. This is fully degenerated space-time inhabited by zero-particles. Regular relativistic mass of zero-particles is zero, but their gravitational rotational mass M is not zero (60). Besides, within the wave-particle dual concept equation of wave phase (eikonal) for zero particles is a standing wave equation (79). In other words from viewpoint of regular observer zero-particles are light-like standing waves (waves of "stopped" light).

As long as gravitational potential w grows we "descend" into the funnel of zero-space deeper and deeper until w finally becomes equal to c^2 and we found ourselves in a black hole (gravitational collapser). The surface of a black hole in zero-space shrinks to a point $g_{ik}dx^i dx^k{=}0$.

It is the matter of degenerated black holes (which contrary to regular black holes are collapsed matter of zero-space) through which interaction between our-world particles and mirror-world particles is possible.

There is another interesting fact. Zero-space can only exist in presence of rotation under condition $w{+}v_i u^i{=}c^2$. In absence of rotation zero-space always collapses, i. e. black hole expands to occupy the whole zero-space. If both gravitational field and rotation are absent penetration inside the membrane becomes impossible and any connection between our world and the mirror world is lost.

The picture we have built so far is a static one. To "vitalize" it we need to look at the nature of the mechanism that "moves the world".

Motion of real particles is determined by their four-dimensional vector of impulse $P^{\alpha}=m_0\frac{dx^{\alpha}}{ds}$, which length is

$$\sqrt{P_{\alpha}P^{\alpha}}=\pm m_0\,. \tag{220}$$

This says that real particle with positive rest-mass $m_0>0$, which exists in regular world with direct flow of time, always has a dual (twin) particle with rest-mass $m_0<0$, which exists in the mirror world. We can suppose that particle with opposite mass "charges" attract each other in the same manner as particles with opposite electric charges.

The condition (220) also means that vector P^{α} is dual by its physical nature and falls apart into two parts, one of which stays in regular world, while another stays in the mirror world. Opposite mass "charges" evolve and move "towards" each other, penetrate zero-space producing standing waves there, and then get into a black hole (each one from its own side). Once they meet in a zero-point ($t_0=0$) they annihilate. Annihilation energy throws them away from the black hole first into zero-space and then into regular world and the mirror world. Then the whole process repeats again and again. Exit of particle into our world is *materialization*, its penetration into zero-space is *dematerialization*. Observed life span of particles is actually the period of time between materialization and dematerialization.

The same is true for particles with zero relativistic masses (light-like particles). Duality of their dynamic vector $K^{\alpha}=\frac{\omega}{c}\frac{dx^{\alpha}}{d\sigma}$ makes its projection onto time sign-alternating

$$\frac{K_0}{\sqrt{g_{00}}}=\pm\frac{\omega}{c}\,. \tag{221}$$

In general, such pure geometric viewpoint allows to see that collapsed particles of black holes, regular particles of zero-space, as well as particles of light-like and substantional worlds are a sequence of states of the same space-time structure.

Epilogue

In *Rainbow Far Away*, written by Arcady and Boris Strugatsky over 30 years ago a character recalls that…

"Being a schoolboy he was surprised by the problem: move things across vast spaces in no time. The goal was set to contradict any existing views of absolute space, space-time, kappa-space… At that time they called it 'punch of Riemannian fold'. Later it would be dubbed 'hyper-infiltration', 'sigma-infiltration', or 'zero-contraction'. At length it was named zero-transportation or 'zero-T' for short. This produced 'zero-T-equipment', 'zero-T-problems', 'zero-T-tester', 'zero-T-physicist'.

— What do you do?

— I'm a zero-physicist.

A look full of surprise and admiration.

— Excuse me, could you explain what zero-physics is? I don't understand a bit of it.

— Well… Neither I do".

This passage might be a good afterword to our study. In the early 1960s words like "zero-space" or "zero-transportation" sounded science-fiction or at least something to be brought to life generations from now.

But science is progressing faster then we think it does. The results obtained in this book suggest that the variety of existing particles and types of their interaction is not limited to those known to contemporary physics. We should expect that further advancement of experimental technique and technology will discover zero-particles, which inhabit degenerated space-time (zero-space) and can be observed as waves of "stopped" light. From viewpoint of a regular observer zero-particles move instantly, thus they can realize zero-transportation.

We think it's a mistake to believe that most Laws of Nature have been already discovered by contemporary science. More likely we are just at the very beginning of a long road to the Unknown World.

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Reference expressions

Components of the operator of projection on time (monad b^α) in an accompanying frame of reference ($b^i=0$)

$$b^0 = \frac{1}{\sqrt{g_{00}}}\,, \quad b_0 = g_{0\alpha}b^\alpha = \sqrt{g_{00}}\,, \quad b_i = g_{i\alpha}b^\alpha = \frac{g_{i0}}{\sqrt{g_{00}}} = -\frac{1}{c}v_i\,,$$

$$\sqrt{g_{00}} = 1-\frac{w}{c^2}\,, \qquad v_i = -c\frac{g_{0i}}{\sqrt{g_{00}}}\,.$$

Components of observable metric tensor $h_{\alpha\beta}$ (the operator of projection on spatial section) in an accompanying frame of reference

$$h_{00} = h_{0i} = 0\,, \qquad h_{ik} = -g_{ik} + \frac{g_{0i}g_{0k}}{g_{00}} = -g_{ik} + \frac{1}{c^2}v_i v_k\,,$$

$$h^{00} = -g^{00} + \frac{1}{g_{00}} = -\frac{1-\frac{1}{c^2}v_i v^i}{1-\frac{w}{c^2}} + \frac{1}{\left(1-\frac{w}{c^2}\right)^2}\,,$$

$$h^{0i} = -g^{0i} = \frac{1}{c\sqrt{g_{00}}}v^i, \qquad h^{ik} = -g^{ik},$$

$$h^0_0 = h^i_0 = 0\,, \qquad h^0_i = \frac{g_{i0}}{g_{00}} = -\frac{1}{c\left(1-\frac{w}{c^2}\right)}v_i\,, \qquad h^i_k = -g^i_k = \delta^i_k\,.$$

Definitions of the velocity of rotation of observer's space

$$v_i = -c\frac{g_{0i}}{\sqrt{g_{00}}}\,, \qquad v^i = -cg^{0i}\sqrt{g_{00}}\,, \qquad v_i = h_{ik}v^k.$$

Derivatives with respect to space-time interval and observable time

$$\frac{d}{ds} = \frac{1}{c\sqrt{1-\mathrm{v}^2/c^2}}\frac{d}{d\tau}\,, \qquad \frac{d}{d\tau} = \frac{^*\partial}{\partial t} + \mathrm{v}^k\frac{^*\partial}{\partial x^k}\,.$$

Chronometrically invariant derivatives

$$\frac{{}^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}\,, \qquad \frac{{}^*\partial}{\partial x^i} = \frac{\partial}{\partial x^i} + \frac{1}{c^2}v_i\frac{{}^*\partial}{\partial t}\,.$$

Relation between determinants of physical observable metric tensor and fundamental metric tensor

$$\sqrt{-g} = \sqrt{h}\,\sqrt{g_{00}}\,.$$

Components of fundamental metric tensor expressed through characteristics of an accompanying frame of reference

$$g_{00} = \left(1 - \frac{w}{c^2}\right)^2, \quad g^{00} = \frac{1 - \frac{1}{c^2}v_i v^i}{1 - \frac{w}{c^2}}\,, \quad g_{0i} = -\frac{v_i}{c}\left(1 - \frac{w}{c^2}\right), \quad g_{ik} = -h_{ik} + \frac{1}{c^2}v_i v_k\,.$$

First and second Zelmanov identities

$$\frac{{}^*\partial A_{ik}}{\partial t} + \frac{1}{2}\left(\frac{{}^*\partial F_k}{\partial x^i} - \frac{{}^*\partial F_i}{\partial x^k}\right) = 0\,,$$

$$\frac{{}^*\partial A_{km}}{\partial x^i} + \frac{{}^*\partial A_{mi}}{\partial x^k} + \frac{{}^*\partial A_{ik}}{\partial x^m} + \frac{1}{2}\left(F_i A_{km} + F_k A_{mi} + F_m A_{ik}\right) = 0\,.$$

Zelmanov relations between regular Christoffel symbols and chronometrically invariant values

$$D_k^i + A_{k\cdot}^{\cdot i} = \frac{c}{\sqrt{g_{00}}}\left(\Gamma_{0k}^i - \frac{g_{0k}\Gamma_{00}^i}{g_{00}}\right),$$

$$F^k = -\frac{c^2\Gamma_{00}^k}{g_{00}}\,, \qquad g^{i\alpha}g^{k\beta}\Gamma_{\alpha\beta}^m = h^{iq}h^{ks}\triangle_{qs}^m\,.$$